INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X

Impact factor: 4.295

(Volume3, Issue1)

Available online at: www.ijariit.com

Anti M-Fuzzy Subrings and its Lower Level M-Subrings

Nanthini .S. P.
Associate Professor, PG and Research Department of Mathematics, Hindusthan College of Arts and Science, Tamil Nadu.
nandhuram70@gmail.com

Munirathinam .C.
M.Phil. Scholar (FT), PG and Research Department of Mathematics, Hindusthan College of Arts and Science, Tamil Nadu.
muni18393@gmail.com

Abstract-- In this paper, we introduce the concept of an anti M-fuzzy Subring of an M-ring and lower level subset of an anti M-fuzzy Subring and discussed some of its properties.

Keywords- M-ring, fuzzy set, anti-fuzzy Subring, anti M-fuzzy Subring of an M-ring, level subset and level M-subrings.

I. INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld [2] gave the idea of fuzzy subgroups. Biswas .R [1] introduced the concept of anti fuzzy subgroups. N. Palaniappan, R. Muthuraj [5] discussed some of the properties of anti fuzzy group and its lower level subgroups. Author N. Jacobson [3] introduced the concept of M-group, M-subgroup. Liu [7] introduced the concept of fuzzy rings in 1982. Mourad Oqla Massa'deh [8] discussed on M- fuzzy subrings. In this paper, we introduce the concept of an anti M-fuzzy subring of an M-ring and lower level subset of an anti M-fuzzy Subring and discussed some of its properties.

II. PRELIMINARIES

This section contains some definitions and results to be used in the sequel.

A) ***Anti fuzzy Subring****:*

Let R be a ring. A fuzzy subset A of R is called an anti-fuzzy subring of R, if $\forall$ x, y $\in$ R.

1. $A(x-y) \leq A(x) \vee A(y)$
2. $A(xy) \leq A(x) \vee A(y)$

B) ***M-Ring****:*

Let R be a ring, M be a set if

1. $mx \in R\ \forall\ x \in R,\ m \in M$.
2. $m(xy) = m(x) + m(y)\ \forall x, y \in R,\ m \in M$.
3. $m(xy) = (mx)y = x(my)\ \forall\ x, y \in R,\ m \in M$. Then R is called an M-ring.

*C) **M- Fuzzy Subring**:* Let R be an M-ring and A be ananti fuzzy subring of R. Then A is called an anti M fuzzy Subring of R if

$\forall$ x$\in$ R and m $\in$M. Then, A (mx) $\leq$ A (x).

III. PROPERTIES OF ANTI M-FUZZY SUBRINGS

In this section, we discuss some of the properties of anti M-fuzzy Subring.

3.1 Theorem

Let S be an M-Subring of an M-ring R. Define a fuzzy subset A of R by

$$A(x) = \begin{cases} t1 & \text{if } x \in S, \\ t2 & \text{otherwise} \end{cases} \qquad \forall x \in R \text{ and } t_1 < t_2,\ t_1, t_2 \in [0, 1].$$

Then A is an anti M-fuzzy Subring of R.

Proof:

Assume that S be an M-Subring of an M-ring R.

Case (i)

If x, y$\in$ S then xy$\in$ S

A(x) = $t_{1,}$ A(y) = $t_1$and A(xy)= t_1

Hence, A (xy) = t_1 = max {A(x), A(y)}

Case (ii)

If x$\notin$ S (or) y$\notin$ S then xy$\notin$ S

$\therefore$ A(x) = t_2 (or) A(y) = $t_2$and A (xy)= t_2

Hence, A (xy)= t_2 = max{A(x), A(y)}

Case (iii)

If x, y$\in$S then x-y $\in$ S

A(x) = $t_{1,}$ A(y)= $t_1$and A(x-y)= t_1

Hence, A(x-y) = t_1 = max {A(x), A(y)}

Case (iv)

If x$\notin$ S(or)y$\notin$ S then x-y$\notin$ S

$\therefore$ A(x)= t_2 (or) A(y)= $t_2$and A(x-y)= t_2

Hence, A(x-y) = t_2 = max {A(x), A(y)}

$\therefore$ A is an anti fuzzy Subring of R.

Next we have to prove that A is an anti M-fuzzy Subring of R

Since, S is an M-Subring of R.
We have mx$\in$ S $\forall$ m$\in$ M and x$\in$ S

A(mx)= t_1 = A(x)
If x$\notin$ S. Then A(mx)= t_2 = A(x)

So, A(mx)≤ A(x)

Hence, A is an anti M-fuzzy Subring of R.

3.2Theorem

The union of any two anti M-fuzzy Subring S of an M-Ring R is always an anti M-fuzzy Subring of an M-ring R.

Proof:

Let R be an M-Ring.

Assume that A and B be any two anti M-fuzzy Subring of R.

To prove: (A ∪ B) is an anti M-fuzzy Subring of R.

First we have to prove that (A ∪ B) is an anti fuzzy Subring of R. Consider,

(i) (A ∪ B)(x-y)= A(x-y) ∪ B(x-y)

≤max{A(x), A(y)}∪max{B(x), B(y)}

=max{A(x)∪B(x), A(y)∪B(y)}

(A ∪ B)(x-y)≤ max {(A ∪ B)(x), (A ∪ B)(y)} (ii) (A ∪ B)(xy)= A(xy) ∪ B(xy)

≤max{A(x), A(y)}∪ max{B(x), B(y)}

=max{A(x)∪B(x), A(y)∪B(y)}

(A ∪ B)(xy)≤ max{(A ∪ B)(x), (A ∪ B)(y)}

∴ (A ∪ B) is an anti-fuzzy Subring of R. Now,

We have to prove that (A ∪ B) is an anti M-fuzzy Subring of R.

Consider,

(A ∪ B)(m(x))= max {A(m(x)), B(m(x))}

≤ max {A(x), B(x)} as A and B are anti M-fuzzy Subring of R. = (A ∪ B)(x)

(A ∪ B)(m(x))≤ (A ∪ B)(x)

∴(A ∪ B) is an anti M-fuzzy Subring of R.

3.3 Theorem

If A is an M-fuzzy subring of an M-ring R iff A^c is anti M-fuzzy subring of R.

Proof:

Suppose A is a M-fuzzy subring of R, ∀ x,y∈ R and m∈ M

Consider,

$A(x-y) \geq \{A(x) \wedge A(y)\}$

$A(x-y) \geq \min\{A(x), A(y)\}$

$\Leftrightarrow \quad 1-A(x-y) \leq 1-\min\{1-A(x), 1-A(y)\}$

$\Leftrightarrow \quad A^c(x-y) \leq 1-\min\{A^c(x), A^c(y)\}$

$\Leftrightarrow \quad A^c(x-y) \leq \max\{A^c(x), A^c(y)\}$

$\Leftrightarrow \quad A^c(x-y) \leq \{A^c(x) \vee A^c(y)\}$

Next,

$A(xy) \geq \{A(x) \wedge A(y)\}$

$A(xy) \geq \min\{A(x), A(y)\}$
$\Leftrightarrow$ $1- A(xy) \leq 1-\min\{1-A(x), 1-A(y)\}$

$A^c (xy) \leq 1- \min\{A^c(x), A^c(y)\}$

$\Leftrightarrow$ $A^c (xy) \leq \max\{A^c(x), A^c(y)\}$
$\Leftrightarrow$ $A^c (xy) \leq \{A^c(x) \vee A^c(y)\}$

Hence, A^c is anti-fuzzy subring of R.

Next we have to prove $A^c (mx) \leq A^c (x)$

Consider, $A(mx) \geq A(x)$
$\Leftrightarrow$ $1-A(mx) \leq 1-A(x)$
$\Leftrightarrow$ $A^c (mx) \leq A^c (x)$

$\therefore A^c$ is an anti M-fuzzy subring of R.

3.4 Theorem

If A is an anti M-fuzzy subring of R, then rAr^{-1} is also an anti M-fuzzy subring of R $\forall r \in R$ and $m \in M$.

Proof:

Assume that A is an anti M-fuzzy subring of R.

First we have to prove that rAr^{-1} is an anti-fuzzy subring of R.

To prove: (i) $rAr^{-1}(x-y) \leq rAr^{-1}(x) \vee rAr^{-1}(y)$
(ii) $rAr^{-1}(xy) \leq rAr^{-1}(x) \vee rAr^{-1}(y)$

(i) $rAr^{-1}(x-y) = A(r^{-1}(x-y)r)$ [$\because$ By defn. of coset]
$\leq A\{(r^{-1}xr) \vee (r^{-1}yr)\}$
$\leq A\{(r^{-1}xr)\} \vee A\{(r^{-1}yr)\}$
$\leq \max\{A\{(r^{-1}xr), A(r^{-1}yr)\}$ [$\because$ By defn.of M-fuzzy subring of R]
$\leq \max\{rAr^{-1}(x), rAr^{-1}(y)\}$
$rAr^{-1}(x-y) \leq rAr^{-1}(x) \vee rAr^{-1}(y)$

[$\because$ By defn. of coset]
[$\because$ By defn.of M-fuzzy subring of R]

(ii) $rAr^{-1}(xy)= A(r^{-1}(xy)r)$
$\leq A\{(r^{-1}xr) \vee (r^{-1}yr)\}$
$\leq A\{(r^{-1}xr)\} \vee A\{(r^{-1}yr)\}$
$\leq \max\{A\{(r^{-1}xr), A(r^{-1}yr)\}$
$\leq \max\{rAr^{-1}(x), rAr^{-1}(y)\}$
$rAr^{-1}(xy) \leq rAr^{-1}(x) \vee rAr^{-1}(y)$

Hence, rAr^{-1} is also an anti fuzzy Subring of R. [$\because$By the definition of coset]
Next we have to prove that rAr^{-1} is an anti M-fuzzy Subring of R.

Consider,

$rAr^{-1}(mx) = A(m(r^{-1}xr))$

$\leq A(r^{-1}Ar)$ as A is anti M-fuzzy Subring

$= rAr^{-1}(x)$

$rAr^{-1}(mx) \leq rAr^{-1}(x)$

Hence, rAr^{-1} is an anti M- fuzzy Subring of R.

3.5 Theorem

The intersection of any two anti M-fuzzy subrings of an M-ring R is always an anti M-fuzzy Subring of an M-ring R.

Proof:

Let R be an M-ring.
Assume that A and B be any two anti fuzzy subrings of R.

To prove: (A∩B) is an anti M-fuzzy Subring of R.
First we have to prove that (A∩B) is an anti fuzzy Subring of R.

Consider,

(i) $(A\cap B)(x-y) = A(x-y)\cap B(x-y)$

$\leq \max\{A(x),\ A(y)\}\cap\max\{B(x),\ B(y)\}$
$= \max\{A(x)\cap B(x), A(y)\cap B(y)\}$

$(A\cap B)(x-y) \leq \max\{(A\cap B)(x), (A\cap B)(y)\}$

(ii) $(A\cap B)(xy) = A(xy)\cap B(xy)$

$\leq \max\{A(x),\ A(y)\}\cap\max\{B(x),\ B(y)\}$
$= \max\{A(x)\cap B(x), A(y)\cap B(y)\}$

$(A\cap B)(xy) \leq \max\{(A\cap B)(x), (A\cap B)(y)\}$

∴(A∩B) is an anti fuzzy Subring of R.

Now, We have to prove that (A∩B) is an anti M-fuzzy Subring of
R. Consider,

$(A\cap B)(m(xy))= \max\{A(m(xy)), B(m(xy))\}$

$\leq \max\{A(xy), B(xy)\}$ as A and B are anti M-fuzzy Subring of R.
$= (A\cap B)(xy)$

$(A\cap B)(m(xy)) \leq (A\cap B)(xy)$

∴(A∩B) is an anti M-fuzzy Subring of R.

IV. PROPERTIES OF LOWER LEVEL SUBSETS OF AN ANTI M-FUZZY SUBRING OF AN M-RING

In this section, we introduce the concept of lower level subset of an anti M-fuzzy Subring of an M-ring and discuss some of its properties.

D) **The lower level subset**: Let A be a fuzzy subset of S. For $t\in [0,1]$, the lower level subset of A is the set,
$A_t = \{x \in S: A(x) \leq t\}$.

4.1 Theorem

Let A be a fuzzy subset of an M-ring R. If A is an anti M-fuzzy Subring of R, then the lower level subsets A_t, $t\in Im(A)$ are M-subrings of R.

Proof:

Given that A is anti M-fuzzy Subring.

To prove: A_t is anti M-fuzzy Subring of R.

First to prove A_t is an anti fuzzy Subring of R.

Let $t \in Im(A)$ and $x, y \in A_t$

We have $A(x) \leq t$, $A(y) \leq t$

Now,

$$A(x-y) \leq \max\{A(x), A(y)\}$$

$$\leq \max\{t, t\}$$

$$\leq t$$

$$A(x-y) \leq t$$

ie.,) $x-y \in A_t$

Moreover, If $x, y \in A_t$

We have,

$A(x) \leq t$, $A(y) \leq t$

$A(xy) \leq \max\{A(x), A(y)\} \leq \max\{t,t\}$

$A(xy) \leq t$

ie.,) $xy \in A_t$

Hence, A_t is an anti fuzzy subring of R.

Now, to prove A_t is an anti M-fuzzy subring of R.

For any $x \in A_t$ and $m \in M$

$$A(mx) \leq A(x)$$

$$\leq t$$

Hence, $A(mx) \leq t$

ie.,) $mx \in A_t$

Hence,

$\therefore A_t$ is an anti M-fuzzy subring of R.

4.2 Theorem

Any M-subringH of an M-ring G can be realized as a lower level M-subring of some anti M-fuzzy subring of R.

Proof:

Let A be a fuzzy subset andx∈ R.

Define,

A(x) 0 x ∈ S
= x ∉ S, where t ∈ 0,1

Prove that A is an anti M-fuzzy subring of R.

First to prove A is an anti fuzzy subring of R.

Case (i)

Suppose x, y∈ S.
Then x∈ S, y∈ S and x-y∈ S

A(x)= 0, A(y)= 0 and A(x-y)= 0

Hence,

A(x-y)≤ max{A(x), A(y)}

Case (ii)

Suppose x∈ S and y∉ S
Then x-y∉ S

A(x)= 0, A(y)= t and A(xy^{-1}) = t

Hence,

(x-y)≤ max{A(x), A(y)}

Case (iii)

Suppose x, y∈ S.
Then x∈ S, y∈ S and xy∈ S

A(x)= 0, A(y)= 0 and A(xy)= 0

Hence,

A(xy)≤ max{A(x), A(y)}

Case (iv)

Suppose x∈ S and y∉ S
Then xy∉ S

A(x)= 0, A(y)= t and A(xy)= t

Hence,

A(xy)≤ max{A(x), A(y)}

∴A is an anti fuzzy subring of R.
Now,∀m∈ M and x∈ S then mx∈ S

∴A(x)= 0 and A(mx)= 0

Hence,

A(xy)≤ A(x)
Now, ∀m∈ M and x∉ S, then mx∉ S (or) mx∈ S
∴ A(x)= t and A(mx)= 0 (or) t.

Hence,

A(mx)≤ A(x)
∴ A is an anti M-fuzzy subring of R.

For this anti M-fuzzy subring,
= S

CONCLUSION

In this paper, we define a new algebraic structure of anti M-fuzzy Subring of an M-ring and lower level subset of an anti M-fuzzy subring and studied some of its properties. Further, we wish to define the anti M-fuzzy normal Subring of an M-ring and its lower level subsets and also the same in Intuitionist fuzzy and other some rings are in progress.

ACKNOWLEDGEMENT

The authors would like to thank the referee for the valuable comments which helped us to improve this manuscript

REFERENCE

[1] Biswas. R, "Fuzzy Subgroups and Anti Fuzzy Subgroups" Fuzzy Sets and Systems, 35 (1990) 121-124.

[2] Rosenfeld, “Fuzzy Groups " J. Math. Anal. Appl, 35 (1971) 512-517.

[3] N. Jacobson," Lectures in Abstract Algebra", (1951) East-West Press.

[4] Das. P.S, “Fuzzy groups and level subgroups”, J.Math.Anal. Appl, 84 (1981) 264-269.

[5] P.Sundararajan, R.Muthuraj, “Anti M-Fuzzy subgroup and its lower level M-subgroups”, J. Com. Appl, volume 26.no.3(2016).

[6] Y.-C. Ren, “Fuzzy ideals and quotient rings”, Fuzzy Math. 4 (1985), 19-26.

[7] Wang-jin Liu, “Fuzzy invariant subgroups and ideals”, fuzzy sets and systems 8(1982), 13-139.

[8] Mourad Oqla Massa’deh, “on M- fuzzy subrings”, J. Math. Sci. 62(2012), 41-49.

[9] L. A. Zadeh, Fuzzy sets, Inform. Control 8 (1965), 338-353.

