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Applying the Fuzzy Critical Path Method to Manufacturing Tugboat

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Abstract: The critical path method is used to date in production operations whose outputs are relatively large complex end items such as highways, buildings, and ships. This paper considers Fuzzy Mathematical Model to find fuzzy critical path, fuzzy project duration in the network. All the activities in the network are represented by trapezoidal fuzzy numbers. Linear programming model is applied to find fuzzy critical path and fuzzy project duration in manufacturing Tugboat (Tugboat is a small ship that exercises vessels by pushing or towing). Finally practical applications shown with comparison.

Keywords: Project network; Trapezoidal fuzzy number; critical path, linear programming.

I. INTRODUCTION

In today's highly competitive business environment, project management's ability to schedule activities and monitor progress strictly in cost, time and performance guidelines is becoming increasingly important to obtain competitive priorities such as on-time delivery and customization. In many situations, projects can be complicated and challenging to manage. Critical Path Method (CPM) is to identify critical activities on the critical path so that resources may be concentrated on these activities in order to reduce the project length time. Further implementation of CPM requires the availability of clear determined time duration for each activity. To deal with real life situations, Zadeh(1965) introduced the concept of fuzzy set. There is always uncertainty in the network planning of time duration of activities. So, fuzzy critical path method introduced in late 1970s. So many approaches are proposed over the past years to find the fuzzy critical path. In 1983, Gazdik developed a fuzzy network of unknown project to estimate the activity durations and used fuzzy algebraic operations to calculate the duration of the project and its critical path. A chapter of the book (Kaufmann and Gupta 1988) is devoted to the critical path method in which activity times are represented by triangular fuzzy numbers. A new methodology to calculate the fuzzy completion project time is presented in the paper (Cahon and Lee 1988). Nasution (1994) proposed a method to compute total floats and the critical paths in a fuzzy project network. Yao and Lin (2000) proposed a method for ranking fuzzy numbers without the need for any assumptions and have used both positive and negative fuzzy values to define ordering and it is applied to fuzzy project network. Dubois et al.(2003) extended the fuzzy arithmetic operational model to compute the latest starting time of each activity in a fuzzy project network. Chen (2007) proposed an approach based on the extension principle and linear programming formulation to critical path analysis in fuzzy project networks. Chen and Hsueh (2008) presented a simple approach to solve the Critical Path Method problems with fuzzy activity times on the basis of linear programming formulation and the fuzzy number ranking method that are more realistic than crisp ones. A new representation for triangular fuzzy numbers was proposed to find critical path (Kumar and Kaur 2010). In 2012 Sanhita Banerjee and Tapan Kumar Roy are studied difuzzification method for generalized trapezoidal fuzzy numbers Based on the operations; some elementary applications on mensuration are numerically illustrated with approximated values. Eloy Vicente et.al. Improved comparative analysis on similarity measure functions (2015). In

2016, Annie Christi and Malini proposed a new solving method for transportation problems by using BCM (Best Candidates Method) and FTP can be converted into a crisp valued TP using the centroid ranking techniques.

II. EXISTING REPRESENTATION OF TRAPEZOIDAL FUZZY NUMBERS

Two Different representations of trapezoidal fuzzy numbers are presented in this section.

General Representation of Trapezoidal Fuzzy Numbers with Membership Function

A fuzzy number $\tilde{A} = (a, b, c, d)$ where a ≥ b ≥ c ≥ d is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } -\infty < x \le a \\ \dfrac{x-a}{b-a}, & \text{if } a < x \le b \\ 1, & \text{if } b < x \le c \\ \dfrac{d-x}{d-c}, & \text{if } c < x \le d \\ 0, & \text{if } d < x < \infty \end{cases}$$

A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number iff $a = 0, b = 0, c = 0, d = 0.$ A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number iff $a \ge 0.$ Two trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ are said to be equal i.e., $\tilde{A} = \tilde{B}$ iff $a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2.$

Representation of (m,n,α,β) Trapezoidal Fuzzy Numbers with Membership Function

A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ may also be represented as $\tilde{A} = (m, n, \alpha, \beta)$, where $m = b,\ n = c, \alpha = b - a, \beta = d - c.$

A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } -\infty < x \le m - \alpha \\ 1 + \dfrac{m-x}{\alpha}, & \text{if } m - \alpha < x \le m \\ 1, & \text{if } m < x \le n \\ \dfrac{n-x}{\beta} + 1, & \text{if } n < x \le n + \beta \\ 0, & \text{if } n + \beta < x < \infty. \end{cases} \qquad \text{Where} \quad \alpha, \beta \ge 0.$$

A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be zero trapezoidal fuzzy number iff $m = 0, n = 0, \alpha = 0, \beta = 0.$ A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be non-negative trapezoidal fuzzy number iff $m - \alpha \ge 0.$ Two trapezoidal fuzzy numbers $\tilde{A} = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{B} = (m_2, n_2, \alpha_2, \beta_2)$ are said to be equal i.e., $\tilde{A} = \tilde{B}$ iff $m_1 = m_2, n_1 = n_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2.$

1. FUZZY CRITICAL PATH ANANLYSIS USING LINEAR PROGRAMMING

The Fuzzy CPM is a network based method designed to assist in the planning, scheduling and control of the project with fuzzy activity times. Its objective is to construct the time scheduling for the project. Two basic results provided by Fuzzy CPM are the total fuzzy duration time needed to complete the project and the fuzzy critical path. One of the efficient approaches for finding the fuzzy critical paths and fuzzy total duration time of project networks is LP. The LP formulation assumes that a unit flow enters the project network at the start node and leaves at the finish node. In this section, the Linear Programming Formulation of Crisp Critical Path problems is reviewed and also the LP formulation of Fuzzy Critical Path problems is proposed in 2013 by K.Usha Madhuri

Linear Programming Formulation of Fuzzy Critical Path Problems

Consider a project network G = < N,E,T> consisting of a finite set N={1,2,…,n} is set of n nodes and E is the set of activities **i-j**. Denote $\tilde{T}_{ij}$ as the time period of activity **i-j**. The Linear Programming formulation of fuzzy Critical Path Problem is

Maximize $\sum_{i-j\in E} \tilde{T}_{ij} \otimes \tilde{x}_{ij}$

Subject to the constraints $\sum_{j:i-j\in E} \tilde{x}_{1j} = \tilde{1}$, $\sum_{i:i-j\in E} \tilde{x}_{ij} = \sum_{j:j-k\in E} \tilde{x}_{jk}$, $i \neq 1, k \neq n$, $\sum_{i:i-n\in E} \tilde{x}_{in} = \tilde{1}$,

Where $\tilde{1}$ = (1,1,1,1) and $\tilde{x}_{ij}$ is a non-negative real number $\forall\, i-j \in E$.

2. ARITHMETIC FUZZY OPERATIONS

In this section, addition and multiplication operations between two trapezoidal fuzzy numbers are reviewed for the existing methods they are explained detail .

1. Arithmetic Operations between (a,b,c,d) Type Trapezoidal Fuzzy Numbers

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$, and (ii) $\tilde{A}_1 \otimes \tilde{A}_2 = (a', b', c', d')$, where a' =minimum of $\{a_1a_2, b_1b_2, c_1c_2, d_1d_2\}$, $b' = b_1b_2$, $c' = c_1c_2$, $d' =$ maximum of $\{a_1a_2, b_1b_2, c_1c_2, d_1d_2\}$.

2. Arithmetic Operations between (m,n,α,β) Type Trapezoidal Fuzzy Numbers

Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ be two trapezoidal fuzzy numbers, then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$, and

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = (m', n', \alpha', \beta')$, where $m' = m_1m_2$, $n' = n_1n_2$, $\alpha' = m' - \min(d')$,and $\beta' = \max(d') - n'$ where

$$d' = \begin{pmatrix} m_1m_2 - m_1\alpha_2 - m_2\alpha_1 + \alpha_1\alpha_2, m_1m_2 + m_1\beta_2 - m_2\alpha_1 + \alpha_1\beta_1, \\ n_1n_2 - n_1\alpha_2 + n_2\beta_1 - \beta_1\alpha_2, n_1n_2 + n_1\beta_2 + n_2\beta_1 + \beta_1\beta_2 \end{pmatrix}.$$

Ranking Function for Trapezoidal Fuzzy Numbers

Ranking Function is the most important to compare the fuzzy numbers to our approach.

$\Re$:F(R)→R, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists.

Let (a,b,c,d) be a trapezoidal fuzzy number then $\Re(a,b,c,d) = \frac{a+b+c+d}{4}$.

Let (m,n,α,β) be a trapezoidal fuzzy number then $\Re(m,n,\alpha,\beta) = \frac{m+n}{2} + \frac{\beta-\alpha}{4}$.

3. NEW REPRESENTATION OF TRAPEZOIDAL FUZZY NUMBERS

The following section presented a new representation of trapezoidal fuzzy numbers, named as New Representation of Trapezoidal Fuzzy Numbers. In this particular representation all the parameters are represented instead of existing representation of trapezoidal fuzzy numbers, and proposed method is applied to find the fuzzy optimal solution of Fuzzy Critical Path problems then the fuzzy optimal solution is same but the total number of constraints, in converted Crisp Linear Programming problem, is less than the number of constraints, obtained by using the existing representation of trapezoidal fuzzy numbers.

1. (a,b,c,d) can be changed in its New representation is $(x,y,\alpha,\beta)_{New}$,where $x=a, y=b, \alpha=b-a\geq 0$ and $\beta=d-c\geq 0$

2. (m,n,α,β) can be changed in its New representation is $(x,y,\alpha,\beta)_{New}$, where $x=m-\alpha,\ y=n+\beta$.

3. It's said to be zero trapezoidal fuzzy number iff $x=0, y=0, \alpha=0, ,\beta=0$.

4. It is said to be non-negative trapezoidal fuzzy number iff $x\geq 0, y\geq 0$.

5. $\tilde{A}=(x_1,y_1,\alpha_1,\beta_1)_{New}$ $\tilde{B}=(x_2,y_2,\alpha_2,\beta_2)_{New,}$ are said to be equal iff $x_1=x_2, y_1=y_2, \alpha_1=\alpha_2, \beta_1=\beta_2$.

Arithmetic Operations between New Type Trapezoidal Fuzzy Numbers

Let $\tilde{A}_1=(a_1,b_1,c_1,d_1)$ and $\tilde{A}_2=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers and $\tilde{A}_1=(x_1,y_1,\alpha_1,\beta_1)_{New}$ $\tilde{A}_2=(x_2,y_2,\alpha_2,\beta_2)_{New}$ be their New representation then the addition and multiplication operations can be represent into the following arithmetic operations:

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (x_1+x_2, y_1+y_2, \alpha_1+\alpha_2, \beta_1+\beta_2)$, and

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = (x_3,y_3,\alpha_3,\beta_3)$, where $x_3 =$ minimum of $\{a_1a_2, b_1b_2, c_1c_2, d_1d_2\}$,

$\alpha_3 = b_1b_2$, $\beta_3 = c_1c_2$, $y_3 =$ maximum of $\{a_1a_2, b_1b_2, c_1c_2, d_1d_2\}$.

Ranking Formula for New Trapezoidal Fuzzy Numbers

The ranking formula for the new representation trapezoidal fuzzy number.

Let $(x,y,\alpha,\beta)_{New}$ be a trapezoidal fuzzy number then $\Re(x,y,\alpha,\beta)_{New} = \frac{x+y}{2} - \frac{\beta-\alpha}{4}$.

4. FUZZY CRITICAL PATH METHOD USING NEW REPRESENTATION TRAPEZOIDAL FUZZ NUMBERS

Step 1: Represent all the parameters of Fuzzy Critical Path problems by a particular type of trapezoidal fuzzy number and formulate the given problem, as proposed in the section 4.3

Step 2: Convert the fuzzy objective function into the crisp objective function form by using appropriate ranking formula.

Step 3: Convert all the fuzzy constraints and restrictions into the crisp constraints and restrictions by using the arithmetic operation.

Step 4: Find the optimal solution of obtained crisp linear Programming problem by using software.

Step 5: Use the crisp optimal solution in Step4 and find the fuzzy optimal solution.

Step 6: Find the fuzzy critical path and corresponding maximum total completion fuzzy time using the fuzzy optimal solution from Step5.

Fuzzy Critical Path Method with $(x,y,\alpha,\beta)_{New}$ Representation Numbers

In this Section all the parameters of Fuzzy Critical Path problems are represented by

$(x, y, \alpha, \beta)_{New}$ type trapezoidal fuzzy numbers then the proposed method are as follows:

Step 1: The trapezoidal fuzzy number be $(\mathbf{x}, \mathbf{y}, \boldsymbol{\alpha}, \boldsymbol{\beta})_{\mathbf{New}}$ type are and $(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})_{New}$ all the parameters represented by $\tilde{T}_{ij}$ and $\tilde{x}_{ij}$ respectively. The linear programming formulation of fuzzy critical path problems proposed in section 4, this can be written as

Maximize $\sum_{i-j \in E} (t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$

Subject to the constraints

$\sum_{j:i-j \in E} (x_{1j}, y_{1j}, \alpha_{1j}, \beta_{1j}) = (1,1,0,0)$, $\sum_{i:i-j \in E} (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{j:j-k \in E} (x_{jk}, y_{jk}, \alpha_{jk}, \beta_{jk}),\ i \neq 1, k \neq n,$

$\sum_{i:i-n \in E} (x_{in}, y_{in}, \alpha_{in}, \beta_{in}) = (1,1,0,0)$, $(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$ is a non negative trapezoidal fuzzy number $\forall\, \mathbf{i} - \mathbf{j} \in E$.

Step 2: Using the ranking formula, presented previously, the Linear Programming of Fuzzy Critical Path problems may be written as

Maximize $\Re\left[\sum_{i-j \in E} (t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})\right]$

$\sum_{j:i-j \in E} (x_{1j}, y_{1j}, \alpha_{1j}, \beta_{1j}) = (1,1,0,0)$, $\sum_{i:i-j \in E} (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{j:j-k \in E} (x_{jk}, y_{jk}, \alpha_{jk}, \beta_{jk}),\ i \neq 1, k \neq n,$

$\sum_{i:i-n \in E} (x_{in}, y_{in}, \alpha_{in}, \beta_{in}) = (1,1,0,0)$, $(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$ is a non negative trapezoidal fuzzy number $\forall\, \mathbf{i} - \mathbf{j} \in E$.

Step 3: Using the arithmetic operation and definitions, Fuzzy Linear Programming problem obtained in Step2 and is converted into the Crisp Linear Programming problem:

Maximize $\Re\left[\sum_{i-j \in E} (t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})\right]$

Subject to the constraints

$\sum_{j:i-j \in E} x_{1j} = 1$, $\sum_{j:i-j \in E} y_{1j} = 1$, $\sum_{j:i-j \in E} \alpha_{1j} = 0$, $\sum_{j:i-j \in E} \beta_{1j} = 0$, $\sum_{i:i-j \in E} x_{ij} = \sum_{j:j-k \in E} x_{jk},\ i \neq 1, k \neq n,$ $\sum_{i:i-j \in E} y_{ij} = \sum_{j:j-k \in E} y_{jk},\ i \neq 1, k \neq n,$

$\sum_{i:i-j \in E} \alpha_{ij} = \sum_{j:j-k \in E} \alpha_{jk},\ i \neq 1, k \neq n,$ $\sum_{i:i-j \in E} \beta_{ij} = \sum_{j:j-k \in E} \beta_{jk},\ i \neq 1, k \neq n,$ $\sum_{i:i-n \in E} x_{in} = 1$, $\sum_{i:i-n \in E} y_{in} = 1$, $\sum_{i:i-n \in E} \alpha_{in} = 0$, $\sum_{i:i-n \in E} \beta_{in} = 0$; x_{ij},

y_{ij}, $\alpha_{ij}, \beta_{ij} \geq 0\ \forall\, \mathbf{i} - \mathbf{j} \in E$.

Step 4: Find the optimal solution $x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}$ by solving the Crisp Linear Programming problem, which is obtained in Step3.

Step 5: Find the fuzzy optimal solution $\tilde{x}_{ij}$ by putting the values of $x_{ij}, y_{ij}, \alpha_{ij}$ and β_{ij} in

$\tilde{x}_{ij} = (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$.

Step 6: Find the maximum total completion fuzzy time by putting the values of $\tilde{x}_{ij}$ in.

Step 7: Determine the fuzzy critical path by combining all the activities **i-j** such that $\tilde{x}_{ij} = (1,1,0,0)$.

5. NUMERICAL EXAMPLE

In real life situations the data is very useful and can apply for some applications. In Hindustan Shipyard Ltd.,(HSL) to prepare the Tugs, Submarines etc., used CPM (Critical Path Method) and PERT(Project Evaluation Review Technique) to find time calculations. To do this work in Fuzzy Environment, the required data is collected from HSL and converted the data into Fuzzy Numbers as trapezoidal form. The problem is to make tugboat and to find the Fuzzy Critical Path i.e. maximum total completion fuzzy time of the

Project network shown in Picture1 and Figure1 respectively, in which activity names and the fuzzy time duration of each activity is represented by the following (a,b,c,d) type (m,n,α,β) type and $(x,y\alpha,\beta)$ trapezoidal fuzzy numbers with activities presented in Table 1 , Table 2, Table 3 and Table 4 respectively .

Picture 1: Picture of a Tug and Figure1: Fuzzy Project Network for Tug

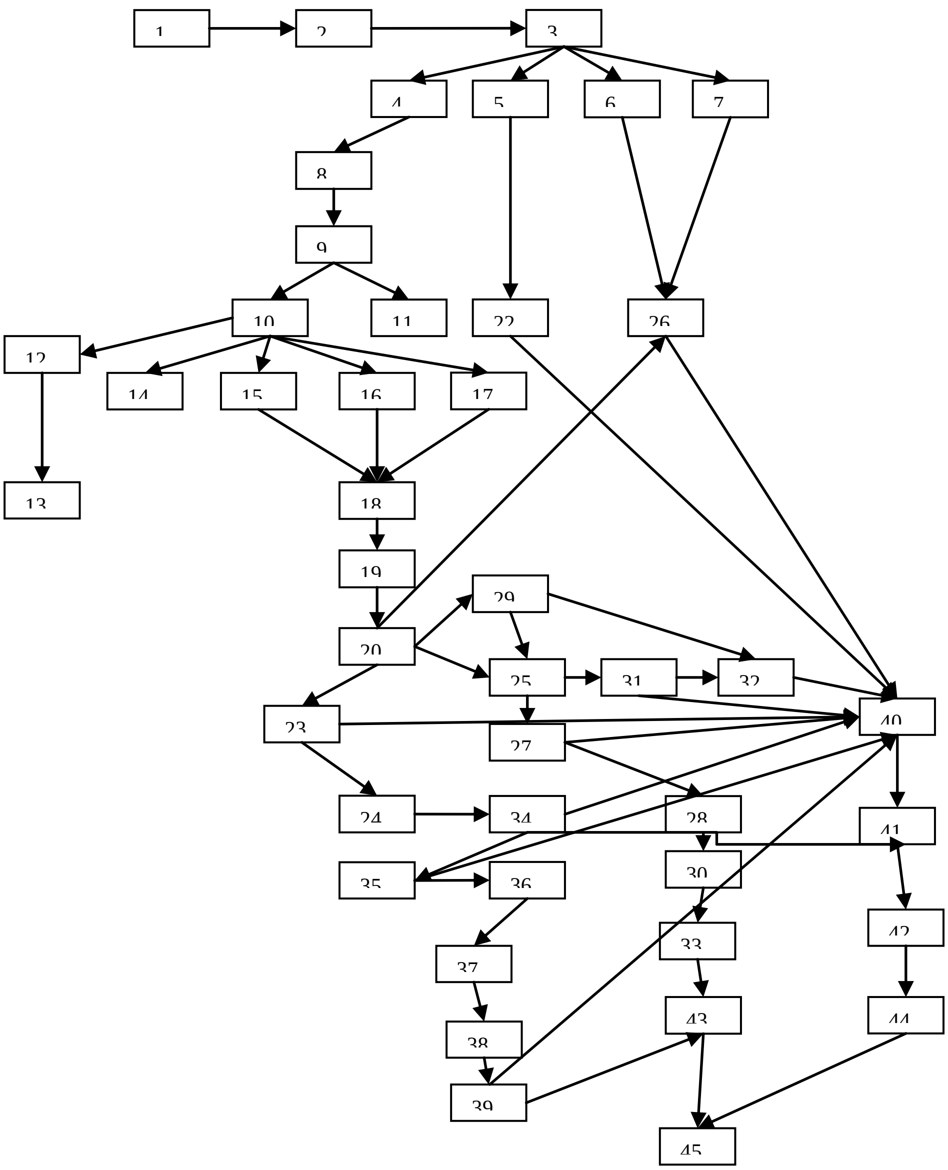

Table 1 : Tugboat Activities

Activity	Activity	Activity
Signing of contract	Completion of underwater fitting works	Installation of Electrical Equipment &Connections
Receipt of approved drawings	Blasting & Painting of Outer hull	Installation of Furniture
Order & receipt of steel	Completion of underwater Painting	
Order & receipt of Main Engine & Aux. Machinery	Launching	AC & Refrigeration works
Order & receipt of Outfit materials	"Erection, Consolidation & survey of Skeg & Docking Pad"	Commissioning of systems pipings
Order & receipt of electrical equipment	Erection & Installation of of Main Engine and Aux. Machinery	commissioning of External fire fighting system
Processing of Steel	Completion of System piping in ER	Commissioning of wheel house equipments
Fabrication of Blocks	Completion of ER Vent works	Installation of navigation & L.S.A eqpt.
Erection and consolidation & Survey of blocks	Erection of super structure decks	Inclining Experiment
Fabrication of super structure	Completion of deck outfit works (Towing /	Basin trials
Erection of Draft marks , Naming Plan, hull paints line etc.	Completion of accommodation works	Sea trials
Erection & Welding of mooring fittings like bollard, fairleads etc.	Accommodation survey	Bollard pull test
Fabrication & Erection of Engine Room Aux. machinery seats	MSB Charging	Final scheme painting
Launching Arrangements (Trigger testing, sliding ways,poppets, cradles etc.	Completion of insulation & Flooring work	Handovering spares & inventory
Pressure Test & Dry Survey of Tanks	Completion of Electrical Cabling works	Delivery

Table 2 : Activities with Time of (a,b,c,d) Type Trapezoidal Fuzzy Numbers

Activity (Name)	Total(No.of days)	Activity (Name)	Total(No.of days)	Activity (Name)	Total(No.of days)
1-2	(0,0,0,0)	14-22	(42,43,44,45)	28-30	(20,22,24,26)
2-3	(1,1,1,1)	15-18	(35,37,38,40)	29-32	(1,1,1,1)
3-4	(10,12,12,14)	16-18	(15,16,17,18)	30-33	(35,36,37,38)
3-5	(13,15,15,17)	17-18	(30,32,33,35)	31-40	(60,62,64,66)
3-6	(75,77,77,79)	18-19	(2,2,2,2)	32-40	(60,62,64,66)
3-7	(75,77,77,79)	19-20	(5,5,5,5)	33-43	(30,32,32,34)
4-8	(75,77,77,79)	20-21	(0,0,0,0)	34-35	(30,32,32,34)
5-22	(350,352,352,354)	20-23	(0,0,0,0)	34-41	(30,32,32,34)
6-26	(300,301,302,303)	20-25	(20,22,24,26)	35-36	(4,4,4,4)
7-26	(300,302,302,304)	20-26	(30,31,32,34)	35-40	(4,4,4,4)
8-9	(70,72,74,76)	20-29	(30,32,34,36)	36-37	(3,3,3,3)
9-10	(120,122,122,124)	21-39	(40,42,44,46)	37-38	(3,3,3,3)
9-11	(135,137,138,139)	22-40	(100,100,100,100)	38-39	(23,24,25,26)
10-12	(100,101,102,103)	23-24	(145,146,147,148)	39-40	(1,1,1,1)
10-14	(105,106,107,108)	23-40	(205,206,207,208)	39-43	(1,1,1,1)
10-15	(110,110,110,110)	24-34	(75,77,77,79)	40-41	(2,2,2,2)
10-16	(110,112,114,116)	25-27	(20,21,22,23)	41-42	(2,2,2,2)
10-17	(120,122,124,126)	25-31	(35,36,37,38)	42-44	(2,2,2,2)
11-25	(45,47,48,50)	26-40	(75,75,75,75)	43-45	(3,3,3,3)
12-13	(10,12,12,14)	27-28	(45,46,47,48)	44-45	(2,2,2,2)
13-20	(35,36,37)	27-40	(45,46,47,48)	45-46	(0,0,0,0)

(Source: Prepared The Activities as (a,b,c,d) type from Crisp Numbers Data from HSL)

The above data is applied to find Fuzzy Optimal Solution with (a,b,c,d)

To compute the fuzzy optimal solution of a project network as shown in Figure is considered and activity time $\tilde{t}_{ij}$ are presented as (a,b,c,d) type trapezoidal fuzzy numbers are presented in Table2.Using the arithmetic operations, described the Fuzzy Linear Programming problem obtained is converted into the Crisp Linear Programming problem. Solving the crisp linear programming problem with software gives the fuzzy optimal solution.

Fuzzy critical path for the fuzzy optimal solution of (a,b,c,d) is

$1\rightarrow2\rightarrow3\rightarrow4\rightarrow8\rightarrow9\rightarrow10\rightarrow15\rightarrow18\rightarrow19\rightarrow20\rightarrow23\rightarrow40\rightarrow41\rightarrow42\rightarrow44\rightarrow45\rightarrow46$.

Replacing the values of x_{ij}, y_{ij}, z_{ij} and γ_{ij}, the maximum total completion fuzzy time is (641,652,656,667).

Table 3: Activities with Time of (m,n,α,β) TypeTrapezoidal Fuzzy Numbers

Table 4 : Activities with Time of (x,y,α,β) TypeTrapezoidal Fuzzy Numbers

Activity (Name)	Total(No.of days)	Activity (Name)	Total(No.of days)	Activity (Name)	Total(No.of days)
1-2	(0,0,0,0)	14-22	(43,44,1,1)	28-30	(22,24,2,2)
2-3	(1,1,0,0)	15-18	(37,38,2,2)	29-32	(1,1,0,0)
3-4	(12,12,2,2)	16-18	(16,17,1,1)	30-33	(36,37,1,1)
3-5	(15,15,2,2)	17-18	(32,33,2,2)	31-40	(62,64,2,2)
3-6	(77,77,2,2)	18-19	(2,2,0,0)	32-40	(62,64,2,2)
3-7	(77,77,2,2)	19-20	(5,5,0,0)	33-43	(32,32,2,2)
4-8	(77,77,2,2)	20-21	(0,0,0,0)	34-35	(32,32,2,2)
5-22	(352,352,2,2)	20-23	(0,0,0,0)	34-41	(32,32,2,2)
6-26	(301,302,1,1)	20-25	(22,24,2,2)	35-36	(4,4,0,0)
7-26	(302,302,2,2)	20-26	(31,32,1,2)	35-40	(4,4,0,0)
8-9	(70,72,74,76)	20-29	(32,34,2,2)	36-37	(3,3,0,0)
9-10	(122,122,2,2)	21-39	(42,44,2,2)	37-38	(3,3,0,0)
9-11	(137,138,2,1)	22-40	(100,100,0,0)	38-39	(24,25,1,1)
10-12	(101,102,1,1)	23-24	(146,147,1,1)	39-40	(1,1,0,0)
10-14	(106,107,1,1)	23-40	(206,207,1,1)	39-43	(1,1,0,0)
10-15	(110,110,0,0)	24-34	(77,77,2,2)	40-41	(2,2,0,0)
10-16	(112,114,2,2)	25-27	(21,22,1,1)	41-42	(2,2,0,0)
10-17	(122,124,2,2)	25-31	(36,37,1,1)	42-44	(2,2,0,0)
11-25	(47,48,2,2)	26-40	(75,75,0,0)	43-45	(3,3,0,0)
12-13	(12,12,2,2)	27-28	(46,47,1,1)	44-45	(2,2,0,0)
13-20	(35,36,37)	27-40	(46,47,1,1)	45-46	(0,0,0,0)

(Source: Prepared The Activities as (m,n,α,β) type from (a,b,c,d) type from Table2)

The above data is applied to find Fuzzy Optimal Solution with (m,n,α,β)

To compute the fuzzy optimal solution of a project network as shown in Figure1 is considered and activity time $\tilde{t}_{ij}$ are presented as (m,n,α,β) type trapezoidal fuzzy numbers are presented in

Table3.Using the arithmetic operations, described the Fuzzy Linear Programming problem obtained is converted into the Crisp Linear Programming problem. Solving the crisp linear programming problem with software gives the fuzzy optimal solution.

The fuzzy optimal solution of (m,n,α,β) and it has the fuzzy critical path is

1→2→3→4→8→9→10→15→18→19→20→23→40→41→42→44→45→46

Replacing the values of x_{ij}, y_{ij}, z_{ij} and γ_{ij}, the maximum total completion fuzzy time is (652,656,11,11).

Activity (Name)	Total(No.of days)	Activity (Name)	Total(No.of days)	Activity (Name)	Total(No.of days)
1-2	(0,0,0,0)	14-22	(42,43,1,1)	28-30	(20,22,2,2)
2-3	(1,1,0,0)	15-18	(35,37,2,2)	29-32	(1,1,0,0)
3-4	(10,12,2,2)	16-18	(15,16,1,1)	30-33	(35,36,1,1)
3-5	(13,15,2,2)	17-18	(30,32,2,2)	31-40	(60,62,2,2)
3-6	(75,77,2,2)	18-19	(2,2,0,0)	32-40	(60,62,2,2)
3-7	(75,77,2,2)	19-20	(5,5,0,0)	33-43	(30,32,2,2)
4-8	(75,77,2,2)	20-21	(0,0,0,0)	34-35	(30,32,2,2)
5-22	(350,352,2,2)	20-23	(0,0,0,0)	34-41	(30,32,2,2)
6-26	(300,301,1,1)	20-25	(20,22,2,2)	35-36	(4,4,0,0)
7-26	(300,302,2,2)	20-26	(30,31,1,2)	35-40	(4,4,0,0)
8-9	(70,72,74,76)	20-29	(30,32,2,2)	36-37	(3,3,0,0)
9-10	(120,122,2,2)	21-39	(40,42,2,2)	37-38	(3,3,0,0)
9-11	(135,138,2,1)	22-40	(100,100,0,0)	38-39	(23,24,1,1)
10-12	(100,102,1,1)	23-24	(145,146,1,1)	39-40	(1,1,0,0)
10-14	(105,107,1,1)	23-40	(206,207,1,1)	39-43	(1,1,0,0)
10-15	(110,110,0,0)	24-34	(75,77,2,2)	40-41	(2,2,0,0)
10-16	(110,114,2,2)	25-27	(20,21,1,1)	41-42	(2,2,0,0)
10-17	(120,124,2,2)	25-31	(36,37,1,1)	42-44	(2,2,0,0)
11-25	(45,48,2,2)	26-40	(75,75,0,0)	43-45	(3,3,0,0)
12-13	(10,12,2,2)	27-28	(45,46,1,1)	44-45	(2,2,0,0)
13-20	(35,36,1,1)	27-40	(46,47,1,1)	45-46	(0,0,0,0)

(Source : Prepared The Activities as (x,y,α,β) from (a,b,c,d) type of Table 3)

The above data is applied to find Fuzzy Optimal Solution with **(a,b,c,d)** and **(m,n,α,β)**

To compute the fuzzy optimal solution of a project network as shown in Figure is considered and activity time $\tilde{t}_{ij}$ are presented as (a,b,c,d) type trapezoidal fuzzy numbers are presented in Table 2 method as follows Using the arithmetic operations, described the Fuzzy Linear Programming problem obtained is converted into the Crisp Linear Programming problem. Solving the crisp linear programming problem with software gives the fuzzy optimal solution.

Fuzzy Optimal Solution Using $(x,y,\alpha,\beta)_{New}$ Representation

In this section, to compute the fuzzy optimal solution considered a project network as shown in Figure and activity time $\tilde{t}_{ij}$ are presented as trapezoidal fuzzy numbers of (x,y,α,β) type trapezoidal fuzzy numbers using (x,y,α,β) representation of presented in Table 4 is method as follows

Step 1: the fuzzy project network shown Figure problem may be formulated as follows

$$\text{Maximize}\{(0,0,0,0)\otimes(x_{12},y_{12},\alpha_{12},\beta_{12})\oplus(1,1,0,0)\otimes(x_{23},y_{23},\alpha_{23},\beta_{23})\oplus(10,12,2,2)\otimes(x_{34},y_{34},\alpha_{34},\beta_{34})\oplus(13,15,2,2)\otimes$$
$$(x_{35},y_{35},\alpha_{35},\beta_{35})\oplus(75,77,2,2)\otimes(x_{36},y_{36},\alpha_{36},\beta_{36})\oplus$$
$$(75,77,2,2)\otimes(x_{37},y_{37},\alpha_{37},\beta_{37})\oplus(75,77,2,2)\otimes(x_{48},y_{48},\alpha_{48},\beta_{48})\oplus(350,352,2,2)\otimes(x_{522},y_{522},\alpha_{522},\beta_{522})\oplus\ldots\oplus(2,2,0,0)\otimes$$
$$(x_{4142},y_{4142},\alpha_{4142},\beta_{4142})\ (2,2,0,0)\otimes(x_{4244},y_{4244},\alpha_{4244},\beta_{4244})\oplus(3,3,0,0)\otimes(x_{4345},y_{4345},\alpha_{4345},\beta_{4345})\oplus(2,2,0,0)\otimes$$
$$(x_{4445},y_{4445},\alpha_{4445},\beta_{4445})\oplus(0,0,0,0)\otimes(x_{4546},y_{4546},\alpha_{4546},\beta_{4546})\}$$

Subject to the constraints

$$(x_{12},y_{12},\alpha_{12},\beta_{12})=(1,1,0,0),$$
$$(x_{23},y_{23},\alpha_{23},\beta_{23})=(x_{12},y_{12},\alpha_{12},\beta_{12})$$

$$\left.\begin{matrix}(x_{34}, y_{34}, \alpha_{34}, \beta_{34}) \oplus (x_{35}, y_{35}, \alpha_{35}, \beta_{35}) \oplus \\ (x_{36}, y_{36}, \alpha_{36}, \beta_{36}) \oplus (x_{37}, y_{37}, \alpha_{37}, \beta_{37})\end{matrix}\right\} = (x_{23}, y_{23}, \alpha_{23}, \beta_{23})$$

$(x_{48}, y_{48}, \alpha_{48}, \beta_{48}) = (x_{34}, y_{34}, \alpha_{34}, \beta_{34})$

$(x_{522}, y_{522}, \alpha_{522}, \beta_{522}) = (x_{35}, y_{35}, \alpha_{35}, \beta_{35})$

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$(x_{4445}, y_{4445}, \alpha_{4445}, \beta_{4445}) = (x_{4244}, y_{4244}, \alpha_{4244}, \beta_{4244})$

$(x_{4546}, y_{4546}, \alpha_{4546}, \beta_{4546}) = (x_{4345}, y_{4345}, \alpha_{4345}, \beta_{4345}) \oplus (x_{4445}, y_{4445}, \alpha_{4445}, \beta_{4445})$

$(x_{4546}, y_{4546}, \alpha_{4546}, \beta_{4546}) = (1,1,0,0)$

$(x_{12}, y_{12}, \alpha_{12}, \beta_{12})$ etc. are non negative trapezoidal fuzzy numbers.

Step 2: Using ranking formula to the Fuzzy Linear Programming problem, formulated in Step1 can be written as
Maximize R {all in step 1 presented} and subject to the constraints.

Step3: Using the arithmetic operations described in previously, the Fuzzy Linear Programming problem obtained in Step2, is converted into the following Crisp Linear Programming problem

Maximize($0\,x_{12} + 0\,y_{12} + 0\,\alpha_{12} + 0\,\beta_{12} + 0.25\,x_{23} + 0.25\,y_{23} + 0\,\alpha_{23} + 0\,\beta_{23} + 2.5\,x_{34} + 3\,y_{34} + 0.5\,\alpha_{34} + 0.5\,\alpha_{34} + 3.25\,x_{35} + 3.75\,y_{35} + 0.5\,\alpha_{35} + 0.5\,\beta_{35} + 18.75\,x_{36} + 19.25\,y_{36} + 0.5\,\alpha_{36} + 0.5\,\beta_{36} + 18.75\,x_{37} + 19.25\,y_{37} + 0.5\,\alpha_{37} + 0.5\,\beta_{37} + 18.75\,x_{48} + 19.25\,y_{48} + 0.5\,\alpha_{48} + 0.5\,\beta_{48} + 87.5\,x_{522} + 88\,y_{522} + 0.5\,\alpha_{522} + 0.5\,\beta_{22} + 75\,x_{626} + 75.25\,y_{626} + 0.25\,\alpha_{626} + 0.25\,\beta_{626} + 75\,x_{726} + 75.5\,y_{726} + 0.5\,\alpha_{726} + \ldots + 0.75\,x_{4345} + 0.75\,y_{4345} + 0\,\alpha_{4345} + 0\,\beta_{4345} + 0.5\,x_{4445} + 0.5\,y_{4445} + 0\,\alpha_{4445} + 0\,\beta_{4445} + 0\,x_{4546} + 0\,y_{4546} + 0\,\alpha_{4546} + 0\,\beta_{4546}$).

Subject to the constraints

$x_{12} = 1,\ y_{12} = 1\ \alpha_{12} = 1\ \beta_{12} = 1$

$x_{23} = x_{12}\ y_{23} = y_{12}\ \alpha_{23} = \alpha_{12}\ \beta_{23} = \beta_{12}$

$x_{34} + x_{35} + x_{36} + x_{37} = x_{23};\ y_{34} + y_{35} + y_{36} + y_{37} = y_{23};\ \alpha_{34} + \alpha_{35} + \alpha_{36} + \alpha_{37} = \alpha_{23};$

$\beta_{34} + \beta_{35} + \beta_{36} + \beta_{37} = \beta_{23}$

$x_{48} = x_{34};\ y_{48} = y_{34};\ \alpha_{48} = \alpha_{34};\ \beta_{48} = \beta_{34}$

$x_{522} = x_{35};\ y_{522} = y_{35};\ \alpha_{522} = \alpha_{35};\ \beta_{522} = \beta_{35}$

$x_{626} = x_{36};\ y_{626} = y_{36};\ \beta_{626} = \beta_{36};\ \alpha_{626} = \alpha_{36}$

$x_{726} = x_{37};\ y_{726} = y_{37};\ \alpha_{726} = \alpha_{37};\ \beta_{726} = \beta_{37}$

$x_{4041} = x_{2640} + x_{2740} + x_{3140} + x_{3240} + x_{3940} + x_{3540};\ y_{4041} = y_{2640} + y_{2740} + y_{3140} + y_{3240} + y_{3940}\ y_{3540}$

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$x_{4546} = x_{4345} + x_{4445};\ y_{4546} = y_{4345} + y_{4445};\ \alpha_{4546} = \alpha_{4345} + \alpha_{4445};\ \beta_{4546} = \beta_{4345} + \beta_{4445}$

$x_{4546} = 1;\ y_{4546} = 1;\ \alpha_{4546} = 0;\ \beta_{4546} = 0.$

Step 4: Solve the crisp linear programming problem using software, obtained in step3 to get the fuzzy optimal solution $x_{12} = y_{12} = \alpha_{12} = \beta_{12} = x_{23} = y_{23} = \alpha_{23} = \beta_{23} = x_{34} = y_{34} = \alpha_{34} = \beta_{34} = x_{48} = y_{48} = \alpha_{48} = \beta_{48} = x_{89} = y_{89} = \alpha_{89} = \beta_{89} = x_{910} = y_{910} = \alpha_{910} = \beta_{910} = x_{1015} = y_{1015} = \alpha_{1015} = \beta_{1015} = x_{1518} = y_{1518} = \alpha_{1518} = \beta_{1518} = x_{1819} = y_{1819} = \alpha_{1819} = \beta_{1819} = x_{1920} = y_{1920} = \alpha_{1920} = \beta_{1920} = x_{2023} = y_{2023} =$

$\alpha_{2023}=\beta_{2023}=x_{2340}=y_{2340}=\alpha_{2340}=\beta_{2340}=x_{4041}=y_{4041}=\alpha_{4041}=\beta_{4041}=x_{4142}=y_{4142}=\alpha_{4142}=\beta_{4142}=x_{4244}=y_{4244}=\alpha_{4244}=\beta_{4244}=x_{4445}=y_{4445}=\alpha_{4445}=\beta_{4445}=x_{4546}=y_{4546}=\alpha_{4546}=\beta_{4546}=1$ and the remaining all values equal to zero.

Step5: By substituting the values x_{ij}, y_{ij}, z_{ij} and γ_{ij} in $\tilde{x}_{ij}=(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$. The optimal solution is $\tilde{x}_{12}$=(1,1,0,0), $\tilde{x}_{23}$ =(1,1,1,1), $\tilde{x}_{34}$=(1,1,0,0), $\tilde{x}_{35}$=0, $\tilde{x}_{36}$=0, $\tilde{x}_{37}$=0, $\tilde{x}_{48}$=(1,1,0,0), $\tilde{x}_{522}$=0,
$\tilde{x}_{626}$=0, $\tilde{x}_{726}$=0, $\tilde{x}_{89}$=(1,1,0,0), $\tilde{x}_{910}$=(1,1,0,0), $\tilde{x}_{911}$=0, $\tilde{x}_{1012}$=0, $\tilde{x}_{1014}$=0, $\tilde{x}_{1015}$=(1,1,0,0), $\tilde{x}_{1016}$=0, $\tilde{x}_{1017}$=0, $\tilde{x}_{1125}$=0, $\tilde{x}_{1213}$=0, $\tilde{x}_{1320}$=0, $\tilde{x}_{1422}$=0, $\tilde{x}_{1518}$=(1,1,0,0), $\tilde{x}_{1618}$=0, $\tilde{x}_{1718}$=0, $\tilde{x}_{1819}$=(1,1,0,0), $\tilde{x}_{1920}$=(1,1,0,0), $\tilde{x}_{2021}$=0, $\tilde{x}_{2023}$=(1,1,0,0), $\tilde{x}_{2025}$=0, $\tilde{x}_{2026}$ =0. $\tilde{x}_{2340}$=(1,1,0,0), $\tilde{x}_{2527}$=0, $\tilde{x}_{2531}$=0, $\tilde{x}_{2740}$=0, $\tilde{x}_{2728}$=0, $\tilde{x}_{2830}$=0, $\tilde{x}_{2932}$=0, $\tilde{x}_{3033}$=0, $\tilde{x}_{3140}$=0, $\tilde{x}_{3240}$=0, $\tilde{x}_{3343}$=0, $\tilde{x}_{3435}$=0, $\tilde{x}_{3441}$=0, $\tilde{x}_{3536}$=0, $\tilde{x}_{3540}$=0, $\tilde{x}_{3637}$=0, $\tilde{x}_{3738}$=0, $\tilde{x}_{3839}$=0, $\tilde{x}_{3940}$=0, $\tilde{x}_{3943}$=0, $\tilde{x}_{4041}$=(1,1,0,0) , $\tilde{x}_{4142}$=(1,1,0,0), $\tilde{x}_{4244}$=(1,1,0,0), $\tilde{x}_{4345}$=0, $\tilde{x}_{4445}$=(1,1,0,0), $\tilde{x}_{4546}$=(1,1,0,0).

Step6: Fuzzy critical path for the fuzzy optimal solution is
1→2→3→4→8→9→10→15→18→19→20→23→40→41→42→44→45→46.

Replacing the values of $x_{ij}, y_{ij}, \alpha_{ij}$ and β_{ij} in Step1

Hence the maximum total completion fuzzy time is (641,652, 11,11).

1. Comparison of New Representation with Existing Representation Fuzzy Numbers

The results of the numerical example are equal with little bit time of difference in real life situation. The fuzzy critical path method using for all types of fuzzy numbers same but there is different between existing method and new representation method. while comparing the number of constraints are very less than in crisp linear programming problem with existing methods. For fast results we may use this method instead of existing representation of trapezoidal fuzzy numbers to get the better improvements for fuzzy optimal solution of fuzzy critical path. The results of representation trapezoidal fuzzy numbers presented in Table 5.

Table 5: Results for Existing Method and Proposed Representation of Fuzzy Numbers

Type of Trapezoidal Fuzzy Numbers	Number of Constraints in Fuzzy Linear programming problem	Number of Constraints in Crisp Linear Programming problem	Maximum Total completion of fuzzy time
(a,b,c,d)	46	(4×46)+(3×63)=184+189=373	(641,652,656,667)
(m,n,α,β)	46	(4×46)+(2×63)=184+189=310	(652,656,11,11)
$(x,y,\alpha,\beta)_{New}$	46	(4×46)=184	(641,652,11,11)

(Source: Results from Representation Methods)

Hence, the Fuzzy Critical Path
1→2→3→4→8→9→10→15→18→19→20→23→40→41→42→44→45→46
For existing methods the total durations is same i.e. 654 days
But to prepare a small Tugboat for existing method total duration is 646.5 days.
one can save the time for preparing 7.5 days.

CONCLUSION

This paper considered Linear Programming fuzzy mathematical model to fuzzy critical path and project duration in the network applying this in manufacturing tugboat. This practical application is shown the advantages of New representation over existing representation of fuzzy numbers. This application problem can be implemented for further results without using software.

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