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ON THE PRINCIPLE OF EXCHANGE OF STABILITIES IN THE MAGNETOHYDRODYNAMIC BENARD PROBLEM WITH VARIABLE GRAVITY BY POSITIVE OPERATOR METHOD.

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Abstract:-In the present paper, the problem of Benard for the magneto hydrodynamic field heated from below with variable gravity is analyzed* and it is established by the method of positive operator of Weinberger *and by using the properties of Green's function* that principle of exchange of stabilities is valid *for this general problem, when g(z) is non-negative throughout the fluid layer.

KEYWORDS: Magneto hydrodynamic field, variable gravity, positive operator, Green's function principle of exchange of stabilities.

I. INTRODUCTION

Rayleigh–Bénard convection is a fundamental phenomenon found in many atmospheric and industrial applications. The problem has been studied extensively experimentally and theoretically because of its frequent occurrence in various fields of science and engineering. This importance leads the authors to explore different methods to study the flow of these fluids. Many analytical and numerical methods have been applied to analyze this problem in the domain of Newtonian fluids, including the linearized perturbation method, the lattice Boltzmann method (LBM), which has emerged as one of the most powerful computational fluid dynamics (CFD) methods in recent years.

A problem in fluid mechanics involving the onset of convection has been of great interest for some time. The theoretical treatments of convective problems usually invoked the so-called principle of exchange of stabilities (PES), which is demonstrated physically as convection occurring initially as a stationary convection. This has been stated as "all non decaying disturbances are non oscillatory in time". Alternatively, it can be stated as "the first unstable Eigen values of the linear zed system have imaginary part equal to zero".

Pellew and Southwell's [l940] celebrated result establishes the validity of the principle of exchange of stabilities for the simple BCnard problem. No such result, however, exists in the magneto hydrodynamic case when a uniform vertical magnetic field opposite to gravity is impressed upon the system and the possibility of the occurrence of oscillatory motions, in this case, has been a matter of speculation only in the literature on the subject. Thompson [1951] investigated this problem for inviscid fluids and derived a sufficient criterion, in terms of the parameters of the system alone, for the validity of this principle but his investigation is somewhat limited on account of the neglect of the effects of viscosity. Chandrasekhar [1952] analysed the same problem and solved the governing equations, which include the viscous effects in a case which he states is appropriate to two free boundaries and concluded the validity of the principle of exchange of stabilities.

Dhiman and Lata [2012] have worked on Positive Operator Method to establish Principle of Exchange of Stabilities in Thermal Convection of Visco elastic Fluid. Lata. [2013] have studied on the principle of exchange of stabilities in thermal instability of Walter's fluid in porous medium with variable gravity by positive operator. In the present paper, the problem of Benard for the magneto hydrodynamic field heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and by using the properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer

II. BASIC EQUATIONS AND BOUNDARY CONDITIONS

The basic equations and boundary conditions, in their non dimensional forms, for the magneto hydrodynamic simple Benard problem when a uniform vertical magnetic field opposite to gravity is impressed upon the system are given by (cf. Chandrasekhar [1961] and Gupta et al. [1983].

$$\left(D^2-k^2\right)\left(D^2-k^2-\frac{\sigma}{p_r}\right)W=Rk^2\theta-QD\left(D^2-k^2\right)h_z \qquad (1)$$

$$\left(D^2-k^2-\sigma\right)\theta=-W \qquad (2)$$

$$\left(D^2-k^2-\frac{\sigma\sigma_1}{p_r}\right)h_z=-DW \qquad (3)$$

W= θ =0 on both the boundaries (4)

$D'W$ =0 a dynamically free boundary (5)

DW=0 on a rigid boundary (6)

h_z =0 on both the boundaries if the regions

Outside the fluid are perfectly conducting (7)

$Dh_z=\mp ah_z$ On both the boundaries if the regions

Outside the fluid are insulating (8)

where z is the vertical coordinate, z = 0 and z = 1 represent the two boundaries, D = d/dz, W is the vertical velocity, θ is the temperature, h_z is the vertical magnetic field, R is the Rayleigh number, k^2 is the square of the wave number, and $\sigma=\sigma_r+i\sigma_i$ is the complex growth rate.

Equations (1) - (3) and appropriately adequate boundary conditions from (4)-(8) pose an eigenvalue problem for σ and we wish to the basic equations and boundary conditions, in their non dimensional
Forms, for the magneto hydrodynamic simple Benard problem

III. THE METHOD *OF POSITIVE OPERATOR*

We seek conditions under which solutions of equations (1)-(3) together with the boundary conditions (4)-(8) grow. The idea of the method of the solution is based on the notion of a 'positive operator', a generalization of a positive matrix, that is, one with all its entries positive. Such matrices have the property that they possess a single greatest positive eigenvalue, identical to the spectral radius. The natural generalization of a matrix operator is an integral operator with non-negative kernel. To apply the method, the resolvent of the linearized stability operator is analyzed. This resolvent is in the form of certain integral operators. When the Green's function Kernels for these operators are all nonnegative, the resulting operator is termed positive. The abstract theory is based on the Krein –Rutman theorem [1962], which states that;

"If a linear, compact operator A, leaving invariant a cone $\hbar$, has a point of the spectrum different from zero, then it has a positive eigen value λ, not less in modulus than every other eigen value, and this number corresponds at least one eigen vector $\phi \in \hbar$ of the operator A, and at least one eigen vector $\varphi \in \hbar^*$ of the operator A^*". For the present problem the cone consists of the set of nonnegative functions.

To apply the method of positive operator, formulate the above equations (1)-(3) together with the boundary conditions (4)-(8) in terms of certain operators as;

$$\left(\tilde{M} + \frac{\sigma}{p_r}\right)\tilde{M}w = g(z)R\theta k^2 + QD\tilde{M}h_z \tag{9}$$

$$\left(\tilde{M} + \sigma\right)\theta = W \tag{10}$$

$$\left(\tilde{M} + \frac{\sigma\sigma_1}{P_r}\right)h_z = DW \tag{11}$$

where, $\tilde{M}w = mw, \quad w \in \mathrm{dom}\tilde{M}; \quad \tilde{M}^2 w = m^2 w, \quad w \in \mathrm{dom}\left(\tilde{M}\tilde{M}\right)$; and $\tilde{M}\theta = m\theta, \quad w \in \mathrm{dom}\tilde{M}$ The domains are contained in B, where $B = L^2(0,1) = \left\{\phi \;\middle|\; \int_0^1 |\phi|^2\, dz < \infty\right\}$,with scalar product $\langle \phi, \varphi \rangle = \int_0^1 \phi(z)\, \overline{\varphi(z)}\, dz$, $\phi, \varphi \in$ B ; and norm $\|\phi\| = \langle \phi, \phi \rangle^{1/2}$.We know that L^2 (0,1) is a Hilbert space, so, the domain of M is

dom $\tilde{M}$ = $\{\phi \in B / \; D\phi, m\phi \in B, \;\; \phi(0) = \phi(1) = 0\}$.

We can formulate the homogeneous problem corresponding to equations (1)-(3) by eliminating θ from (9) - (11) as;

$$w = \tilde{M}^{-1}\left(M+\frac{\sigma}{p_r}\right)\left[k^2R\left(\tilde{M}+\sigma\right)^{-1}+QD^2M\left(M+\frac{\sigma\sigma_1}{p_r}\right)^{-1}\right]g\left(z\right) \quad (12)$$

or $\mathrm{w} = \mathrm{K}(\sigma)\mathrm{w}$, (13)

where, $$\mathrm{K}(\sigma) = \tilde{M}^{-1}\left(M+\frac{\sigma}{p_r}\right)\left[k^2R\left(\tilde{M}+\sigma\right)^{-1}+QD^2M\left(M+\frac{\sigma\sigma_1}{p_r}\right)^{-1}\right]g\left(z\right) \quad (14)$$

But $T\left(\sigma\right)=\left(\tilde{M}+\sigma\right)^{-1}$ and exists for $\sigma \in T_k = \left\{\sigma \in C \;\middle|\mathrm{Re}\left(\sigma\right) > -k^2, \mathrm{Im}\left(\sigma\right)=0\right\}$ and $\left\|T\left(\sigma\right)\right\|^{-1} > \left|\sigma+k^2\right|$ for Re $\left(\sigma\right) > -k^2$.

$$T\left(\sigma\right) f = \int_0^1 g\left(z,\xi;\sigma\right) f\left(\xi\right) d\xi ,$$

where, $g(z,\xi,\mathrm{E\,Pr}\,\sigma)$ is Green's function kernel for the operator $\left(M+\sigma\right)$ and is given as

$$g\left(z,\xi,\sigma\right) = \frac{\cosh\left[r\left(1-\left|z-\xi\right|\right)\right]-\cosh\left[r\left(-1+z+\xi\right)\right]}{2r\sinh r}$$

where, $r=\sqrt{k^2+\sigma}$.

In particular, taking $\sigma = 0$, we have $\mathrm{M}^{-1} = \mathrm{T}(0)$ is also an integral operator.

$\mathrm{K}(\sigma)$ defined in (14), which is a composition of certain integral operators, is termed as linearized stability operator. $\mathrm{K}(\sigma)$ depends analytically on σ in a certain right half of the complex plane. It is clear from the composition of K(σ) that it contains an implicit function of σ .

We shall examine the resolvent of the K(σ) defined as $[\mathrm{I}-\mathrm{K}(\sigma)]^{-1}$

$$[\mathrm{I}-\mathrm{K}(\sigma)]^{-1} = \left\{\mathrm{I}-[\mathrm{I}-\mathrm{K}(\sigma_0)]^{-1}[\mathrm{K}(\sigma)-\mathrm{K}(\sigma_0)]\right\}^{-1}[\mathrm{I}-\mathrm{K}(\sigma_0)]^{-1} \quad (15)$$

If for all σ_0 greater than some a,

(1) $[\mathrm{I}-\mathrm{K}(\sigma_0)]^{-1}$ is positive,

(2) $\mathrm{K}(\sigma)$ has a power series about σ_0 in $(\sigma_0-\sigma)$ with positive coefficients; i.e., $\left(-\frac{d}{d\sigma}\right)^n \mathrm{K}(\sigma_o)$ is positive for all n. Hence, we may apply the methods of Weinberger (1969) and Rabinowitz (1969), to show that there exists a real eigenvalue σ_2 such that the spectrum of $\mathrm{K}(\sigma)$ lies in the set $\{\sigma : \mathrm{Re}(\sigma) \le \sigma_2\}$. This is result is equivalent to PES, which was stated earlier as "the first unstable eigenvalue of the linearized system has imaginary part equal to zero."

IV. THE PRINCIPLE OF EXCHANGE OF STABILITIES (PES)

It is clear that $\mathrm{K}(\sigma)$ is a product of certain operators. Condition (1) can be easily verified by following the analysis of Herron [2000, 2001] for the present operator $\mathrm{K}(\sigma)$. The operator $\tilde{\mathrm{M}}^{-1} = \mathrm{T}(0)$ is an integral operator whose Green's function $g(z,\xi;0)$ is nonnegative so $\tilde{\mathrm{M}}^{-1}$ = T(0) is a positive operator. It is mentioned above that $T(\sigma)$ is an integral operator its Green's function kernel $g(z,\xi,\sigma)$ is the Laplace transform of the Green's function $G(z,\xi;t)$ for the boundary value problem

$$\left(-\frac{\partial^2}{\partial z^2}+k^2+\frac{\partial}{\partial t}\right)G=\delta(z-\xi,t), \qquad (17)$$

Where , $\delta(z-\xi,t)$ is Dirac –delta function in two-dimension, with boundary conditions

G$(0,\xi;t)$ =0= G$(1,\xi;t)$ and initial condition $G(z,\xi;0)=0$

Following Herron [2000], by direct calculation of the inverse Laplace transform, we can have

$$\left(-\frac{d}{d\sigma}\right)^n g(z,\xi,\sigma)=\int_0^\infty t^n e^{-\sigma t}\, G(z,\xi,t)\, dt \ \ge 0 \ \text{ for all n and } \sigma_0 > -k^2.$$

$T(\sigma)$ is positive operator for all real $\sigma_0 > -k^2$, and that $T(\sigma)$ has a power series for all real $\sigma_0 > -k^2$. On the same lines we can prove that $T\left(\frac{\sigma}{p_r}\right)$ and $T\left(\frac{\sigma\sigma_1}{p_r}\right)$ are positive.

It has been demonstrated that all of the terms in $\mathrm{K}(\sigma)$ determine positive operator. i.e. $\mathrm{K}(\sigma)$ is a linear, compact integral operator. Thus, $\mathrm{K}(\sigma)$ is a positive Moreover, for σ real and sufficiently large, the norms of the operators $T\left(\frac{\sigma}{p_r}\right)T(0)$, $T\left(\frac{\sigma\sigma_1}{p_r}\right)$ become arbitrarily small. So, $\|\mathrm{K}(\sigma)\|<1$. Hence, $[\mathrm{I}-\mathrm{K}(\sigma)]^{-1}$ has a convergent Neumann series, which implies that $[\mathrm{I}-\mathrm{K}(\sigma)]^{-1}$ is a positive operator. This is the content of condition (1). To verify condition (2) for g(z)≥ 0 for all $z\in[0,1]$ and $\sigma_0 > -k^2$ }

, while k^2 ,R and Q are clearly positive. Therefore by the product rule for differentiation, one concludes that $K(\sigma)$ in (12) satisfies condition (2).

CONCLUSION*:*

In the present paper, principle of exchange of stabilities valid for this general problem, when g(z) is non-negative throughout the fluid layer.

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