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On PARAHYPONORMAL and QUASI PARAHYPONORMAL OPERATORS

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Abstract- In this paper we discuss about a new class of operators on Hilbert space. We call these operators as parahyponormal operators. Moreover, here we discussed some results on quasi parahyponormal and M-Quasi parahyponormal operators.

Keywords: Hilbert space, Normal operator, Self adjoint, Isometry, Parahyponormal operators and quasi parahyponormal operators.

I. INTRODUCTION

Let H denotes the Hilbert Space and L (H) denotes the space of all bounded linear operators defined in the Hilbert space. All hypo normal operators are paranormal Furuta [1] has defined a bounded linear operator T on a Hilbert space H as paranormal if $\|T^2x\| \le \|Tx\|^2$ for every unit vector x in H. The following inclusion denotes the relation between classes of operators.

Hyponormal $\Longrightarrow$ paranormal $\Longrightarrow$ parahyponormal $\Longrightarrow$ quasi parahyponormal

II. DEFINITION

A) *Parahyponormal:*

An operator $T \in L(H)$ is said to be Parahyponormal if $\|Tx\|^2 \le \|TT^*x\|$ for every unit vector x in H with $\|x\| = 1$. As in [2] T is said to be parahyponormal operator if and only if $(TT^*)^2 - 2\lambda(T^*T) + \lambda^2 \ge 0$ for every $\lambda > 0$

B) *Quasi parahyponormal:*

An operator $T \in L(H)$ is said to be quasi parahyponormal if $\|TT^{*}x\|^{2} \leq \|T^{2}T^{*2}x\|$ for every unit vector x in H. As in [3] T is said to be quasi parahyponormal if and only if $\left(T^{2}T^{*2}\right)^{2} + 2\lambda\left(TT^{*}\right)^{2} + \lambda^{2} \geq 0$ for all $\lambda > 0$

C) *M-Quasi* ***parahyponormal:***

An operator $T \in L(H)$ is said to be M-Quasi parahyponormal operator if $\|TT^{*}x\| \leq M\,\|T^{2}T^{*2}x\|\|x\|$ for each unit vector x in H. As in [3] T is said to be M-Quasi parahyponormal if and only if M $\left(T^{2}T^{*2}\right)^{2} + 2\lambda\left(TT^{*}\right)^{2} + \lambda^{2} \geq 0$ for all $\lambda > 0$

When M=1 this operator is called Quasi parahyponormal.

III RESULTS

A) PARAHYPONORMAL

Theorem3.1

Every normal operator T in $B(H)$ is Parahyponormal operator.

Proof

Normal operator is given by, $TT^{*} = T^{*}T$

$$\|Tx\|^{2} = \langle Tx, Tx\rangle$$

$$= \langle T^{*}(Tx), x\rangle$$

$$\leq \|TT^{*}x\|\|x\|$$

$$= \|TT^{*}x, x\| \text{ With } \|x\| = 1$$

$$\leq \|TT^{*}x\|$$

B) QUASI PARAHYPONORMAL

Theorem **3.2**

If a quasi parahyponormal operator T commutes with an isometric operator S, TS is quasi parahyponormal operator.

Proof:

Let A=TS, for all real number λ

$$(\mathrm{A}^{2}A^{*2})^{2} + 2\lambda(\mathrm{AA}^{*})^{2} + \lambda^{2} \geq 0$$

$$\left[(TS)^{2}(TS)^{*2}\right]^{2} + 2\lambda\left[(TS)(TS)^{*}\right]^{2} + \lambda^{2} \geq 0$$

$$\left[\left(T^{2}S^{2}\right)\left(S^{*2}T^{*2}\right)\right]^{2} + 2\lambda\left[(TS)\left(S^{*}T^{*}\right)\right]^{2} + \lambda^{2} \geq 0$$

$$T^{2}S^{2} = S^{2}T^{2}, S^{*2}S^{2} = I, TS = ST, SS^{*} = I$$

$$\left[\left(S^{2}T^{2}\right)\left(S^{*2}T^{*2}\right)\right]^{2} + 2\lambda\left[(ST)\left(S^{*}T^{*}\right)\right]^{2} + \lambda^{2} \geq 0$$

$$(T^{2}T^{*2})^{2} + 2\lambda(\mathrm{TT}^{*})^{2} + \lambda^{2} \geq 0$$

A is quasi parahyponormal operator.

Theorem 3.3.

If a quasi parahyponormal operator T commutes with an isometric operator S, then $\frac{T}{S}$ is quasi parahyponormal operator.

Proof

Let $A=\frac{T}{S}$, we have for any real number λ, $(A^2 A^{*2})^2+2\lambda(AA^*)^2+\lambda^2\geq 0$

$$\left[\left(\frac{T}{S}\right)^2\left(\frac{T}{S}\right)^{*2}\right]^2+2\lambda\left[\left(\frac{T}{S}\right)\left(\frac{T}{S}\right)^*\right]^2+\lambda^2 I\geq 0$$

$$\left[\left(T^2S^{-2}\right)\left(T^{*2}S^{-*2}\right)\right]^2+2\lambda\left[\left(TS^{-1}\right)\left(T^*S^{-*1}\right)\right]^2+\lambda^2 I\geq 0$$

But $T^2S^2=S^2T^2$ and $S^2S^{-*2}=I$, $SS^{-1}=I$

$$\left[\left(S^{-2}T^2\right)\left(S^{-*2}T^{*2}\right)\right]^2+2\lambda\left[\left(S^{-1}T\right)\left(S^{-*1}T^*\right)\right]^2+\lambda^2 I\geq 0$$

$$(T^2T^{*2})^2+2\lambda(TT^*)^2+\lambda^2\geq 0$$

A is quasi parahyponormal operator.

Theorem 4.3.

Let $T\in B(H)$ be an operator of quasi parahyponormal. Then if T is unitary equivalent to S, then S is of quasi parahyponormal.

Proof

Consider $$(T^2T^{*2})^2+2\lambda(TT^*)^2+\lambda^2\geq 0$$

$$(S^2S^{*2})^2+2\lambda(SS^*)^2+\lambda^2\geq 0$$

Put $S=UTU$

Then $$\left[(UTU)(UT^*U)(UTU)(UT^*U)\right]^2+2\lambda\left[(UTU)(UT^*U)\right]^2+\lambda^2 I\geq 0$$

$$\left[UTTT^*T^*U^*\right]^2+2\lambda\left[UTT^*U^*\right]^2+\lambda^2 I\geq 0$$

$$U\left[TTT^*T^*\right]^2U^*+2\lambda U\left[TT^*\right]^2U^*+\lambda^2 I\geq 0$$

$$U\left[(T^2T^{*2})^2+2\lambda(TT^*)^2+\lambda^2 I\right]U^*\geq 0$$

$$(T^2T^{*2})^2+2\lambda(TT^*)^2+\lambda^2\geq 0$$

Theorem.4.4.

If M is a closed subspace of H such that M reduces T then $\left(\frac{T}{M}\right)$ is quasi parahyponormal operator.

Proof

$$\left\|TT^*x\right\|^2\leq\left\|T^2T^{*2}x\right\|$$

$$TP=PTP$$

$$\left\|\left(\frac{T}{M}\right)\left(\frac{T}{M}\right)^{*}x\right\|^{2}=\left\|\left(PTPx\right)\left(PT^{*}Px\right)\right\|^{2}$$
$$=\left\|\left(TPx\right)\left(T^{*}Px\right)\right\|^{2}$$
$$\leq\left\|TT^{*}x\right\|^{2}\leq\left\|T^{2}T^{*2}x\right\|$$
$$=\left\|\left(\frac{T}{M}\right)^{2}\left(\frac{T}{M}\right)^{*2}x\right\|$$

Thus for $x\in M$, $\frac{T}{M}$ is quasi parahyponormal operator.

C) M- QUASI PARAHYPONORMAL

Theorem 4.5

If a M parahyponormal operator double commutes with a hypo normal operator S, then the product TS is M quasi parahyponormal operator.

Proof:

$Let\left\{E\left(t\right)\right\}$ be the resolution of the identity for the self-adjoint operator $S^{*}S$.

Thus $T^{*}T$ and $T^{*2}T^{2}$ both double commutes with every $E\left(t\right)$.

Since S is hyponormal.

we have $S^{*}S\geq SS^{*}$

$$M^{2}\left[\left(TS\right)^{2}\left(T^{*}S^{*}\right)^{2}\right]^{2}+2\lambda\left[\left(TS\right)\left(TS\right)^{*}\right]^{2}+\lambda^{2}$$
$$=M^{2}\left[\left(T^{*2}T^{2}\right)\left(S^{*2}S^{2}\right)\right]^{2}+2\lambda\left[\left(T^{*}T\right)\left(S^{*}S\right)\right]^{2}+\lambda^{2}$$
$$\geq M^{2}\left[\left(T^{*2}T^{2}\right)\left(S^{*}S\right)^{2}\right]^{2}+2\lambda\left[\left(T^{*}T\right)\left(S^{*}S\right)\right]^{2}+\lambda^{2}\text{ [Since }S^{*}S=I\text{]}$$
$$=\int_{0}^{\infty}t^{2}(M\left(T^{*2}T^{2}\right)^{2}+2\lambda\left(T^{*}T\right)^{2}+\lambda^{2})dt$$
$$\geq 0$$

Thus T is M quasi parahyponormal operator.

IV CONCLUSION

A study on Hilbert space operators is very significant in analysis and application. In this work we conclude that the conditions of paranormal are satisfied for the parahyponormal and quasiparahyponormal. We extend it to other operators in future.

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