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Comparison of State Observer Design Algorithms for DC Servo Motor Systems

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Abstract— a state observer is a system that models a real system in order to provide an estimate of the internal state of the system. The design techniques and comparison of four different types of state observers are presented in this paper. The considered observers include Luenberge observer, unknown input observer and sliding mode observer. The application of these observers to a Multiple Input Multiple Outputs (MIMO) DC servo motor model and the performance of observers is assessed. In order to evaluate the effectiveness of these schemes, the simulated results on the position of DC servo motor in terms of residuals including white noise disturbance and additive faults are compared.

Keywords— Luenberger Observer, SMO, UIO.

1. **Introduction**

In science and engineering problems, state space realizations are often used to model linear [1,2] and nonlinear dynamical systems. Some of the examples are system monitoring, state feedback control, and fault detection [14] In all these applications, complete state vector information is necessary to implement the state feedback. However, in practice, all the states are usually not available for feedback because it is often expensive and impractical to have sensors for every state variable, so some form of reconstruction of state variables is required from the measured output. In such scenario, state observer, a computer implemented mathematical model, can be constructed using the mathematical model of the system and the available outputs to obtain an estimate of the true state[4] , and this estimate can then be used as a substitute for . In the classical approach, the reconstruction of state variables comprises the study of unforced system subjected to nonzero initial conditions, and calculation of observer gain to stabilize the error dynamics, hence achieving asymptotic convergence. Since it is difficult to incorporate the modeling errors in the design of observers for linear systems, much attention has been paid on this area in the recent years [15] particularly in connection with the problem of residual generation in observer

2. Modeling of a DC Motor

A DC motor is a second order system with multiple input and multiple outputs. The model is designed according to the parameters, armature resistance, armature inductance, magnetic flux, and voltage drop factor, inertia constant and viscous friction. It is studied as a linear system. The inputs are the armature voltage *UA* (*t*) and the load torque *ML* (*t*). In simulation the armature voltage is given as a step function while the load torque is given a fixed value of 0.1. The measured output signals are the armature current *IA* (*t*) and the speed of the motor ω(*t*). Fig. 1 shows the model of the DC motor.

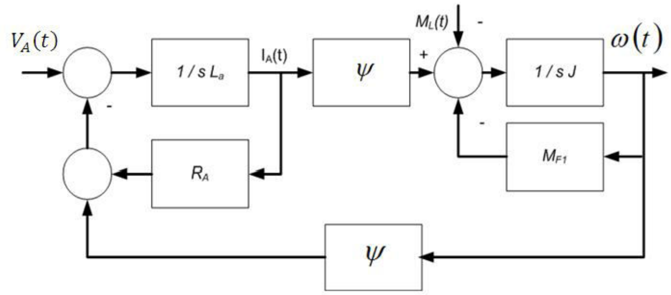

Fig 1 Model Of a DC Motor

The values of parameters are as follows. Armature resistance *Ra* = 1.52 Ω. Armature inductance *La* = 6.82_10^{-3} Ω *s*. Magnetic flux ψ = 0.33 *V s*. Inertia constant *J* = 0.0192 *kg m*2. Viscous friction *MFl* = 0*:*36 *x*10^{-3} *Nms*.

The armature current *IA*(*t*) and armature speed ω(*t*) are represented as in the following-

$$L_aI_a=R_aI_a(t)=\Psi w(t)=V_a(t) \qquad (1)$$
$$Jw(t)=\Psi I_a(t)-M_{FL}w(t)\_M_L(t) \qquad (2)$$

The general continuous state space form with faults or disturbance is represented as

$$x(t)= Fx(t)+Gu(t)+L_lF_l(t) \quad (3)$$
$$y(t)=cx(t)+Du(t)+M_mF_m(t) \quad (4)$$

where $x(t)ЄR^n$ is a state vector, $u(t)ЄR^m$ is a control input vector, $y(t)ЄR^p$ is a measurement output vector, A, B, *C* and *D* are known constant system matrices. The continuous time system in Equations (3) and (4) can be discretised using sampling time to obtain the discrete time model as represented in Equations

$$x(k+1)=Ax(k)+Bu(k)=Lf_l\ (k) \qquad (5)$$
$$y(k)=Cx(k)+Du(k)=MF_m(k) \qquad (6)$$

Where *T* is the sampling time, $L = T*A*L_l$ and $M = M_m$

3. Model of Observer

3.1 Luenberger observer

The analytical redundancy is using mathematical method to reconstruct a model [12] which is under monitoring online. Its main idea is checking the consistency about the actual model with the reconstruct one. The difference or inconsistency between actual [7] and reconstruct model is called the residual. The residual is computed from the observables, which includes measurement values in plants, variables of outputs, and measured inputs. When actual model differ away from ideal state, the residual becomes non-zero, and this would happened by faults [9], besides, noise, disturbance, and other unknown inputs might also cause the residual vary from zero. Generating the residual is the fundamental feature of fault detection technique. In order to get the residual, which indicates the presence or absence of a fault [13] it is necessary to design a residual generator. A residual generator is a crucial role in fault detection system. It main objective is creating a signal which sensitive to fault and insensitive to unknown inputs. Among a variety of techniques, the observed-based method is one of the most common ways [10] in model-based fault diagnosis field. The observer measures the internal state of variable, and it is designed by the modern control theory and mathematical technique. The fault detection filter is one of the observers that can generate the residual. Beard and Jones invented this observer-based generator in 1970s. Their work strengthens the development of model-based fault diagnosis technique [11]. The core of fault detection filter is a state observer, which is based on the nominal system.

For Continuous time linear systems—

$$x(t)= Ax(t)+Bu(t) \qquad (7)$$

$$y(t)= Cx(t) \qquad (8)$$

where $x Є R^n$, $u Є R^m$, $y Є R^r$, a linear system observer equations is given by

$$x^*(t)=Ax(t)+Bu(t)+L[y(t)-y^*(t)] \qquad (9)$$

$$y(t)=Cx^*(t) \qquad (10)$$

Observer error is defined as—

$$e=x-x^* \qquad (11)$$

Substituting above equations-

$$e^*=Ax+Bu-[Ax^*+Bu+L(y-y^*)] \qquad (12)$$

$$\text{Or, } e^*=A(x-x^*)-L(y-y^*)$$

With the help of equation 8, we can say-

$$e^*= (A-LC)e \qquad (13)$$

From equation 8,

$$e(t)=e^{(A-LC)}e(0) \qquad (14)$$

The eigenvalues of the matrix $A \rightarrow LC$ can be made arbitrary by appropriate choice of the observer gain L, when the pair $[A;C]$ is observable (i.e. observability condition holds). So the observer error $e \rightarrow \alpha$ when $t \rightarrow \alpha$

The given system is observable if and only if the $n \times n$ matrix, $[C^T\ A^T C^T : : : (A^T)^{n-1}\ C^T]$ is of rank n. This matrix is called the observability matrix.

3.2 Unknown Input Observer

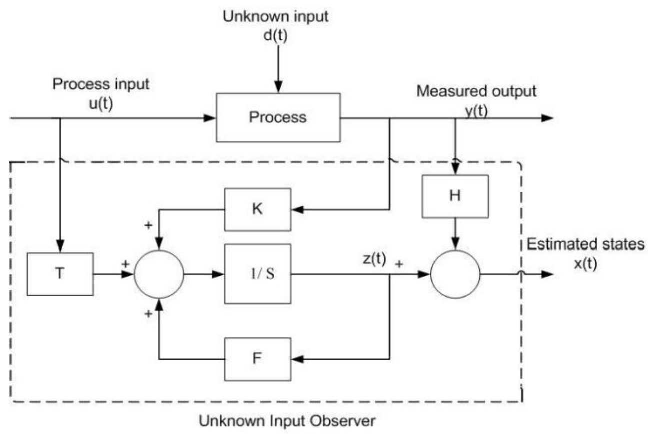

Fig 2- Block diagram of UIO

Consider a continuous linear time invariant space model of the system,

$$x(t)= Ax(t)+Bu(t)+Ed(t) \qquad (15)$$
$$y(t)= Cx(t)$$

$X \in R^{nx1}$ represents the state vector, *u* represents input vector, *y* represents sensor output, *A* represents system coefficient matrix, *B* represents input coefficient matrix, *C* represents output coefficient matrix, $d \in R^{qx1}$ represents the unknown input vector, and $E \in R^{nxq}$ represents the unknown input distribution matrix. The structure of the UIO can be described as,

$$e(t)= x(t)-x^*(t) \qquad (16)$$

The state estimation error can be represented by the following equation,

$$e(t)= x(t)-\hat{x}x^*(t)=x(t)- z(t) -Hy(t)= (1-HC)x(t)-Z(t) \qquad (17)$$

Using Equation 17, derivative of error vector,

$$e^*=(A-HCA-K_1C)e(t)+(A-HCA-K_1C)z(t)+(A-HCA-K_1C)Hy(t)+(1-HC)Bu(t)+(1-HC)Ed(t)-Fz(t)-TBu(t)-K_2y(t) \qquad (18)$$

$$=(A-HCA-K_1C)e(t)+[F-(A-HCA-K_1C)]z(t)-[K_2-(A-HCA-K_1C)]Hy(t)-[T-(1-HC)]Bu(t)- [(1-HC)]Ed(t) \qquad (19)$$

The following relations hold true,

$$(HC-1)E=0 \qquad (20)$$

$$T= 1-HC \qquad (21)$$

$$F=A-HCA-K_1CK_2 \qquad (22)$$

$$K_2=FH \qquad (23)$$

$$K= K_1+ K_2 \qquad (24)$$

Derivative of the error vector is $\dot{e}$ (t) =Fe (t) Hence, the solution of the error vector will be e (t) = e (0). If *F* is chosen as a Hurwitz matrix [3,5] the solution of the error equation goes to zero asymptotically. Therefore, $\hat{x}$ converges to x. Necessary and sufficient conditions for the observer to be a UIO for the system defined in Equations (8) and (15) are

1. *rank* (*CE*) = *rank* (*E*)
2. (*C,A1*) is a detectable pair,

Where $A1=A - E\,[(CE)^T CE]^{-1}(CE)^T CA.$

4. SLIDING MODE OBSERVERS

Sliding mode observers are different from traditional observers mainly due to the injection of non-linear discontinuous term into observer based on the output estimation error. One of the main advantages of using sliding mode observers over their linear counterparts is that during sliding phase, they are insensitive to the unknown inputs. Moreover, they can also be used to reconstruct unknown inputs which could be a combination of system disturbances, faults or nonlinearities. In the design of a sliding mode observer, first a sliding manifold [x|σ(x) =0] is to be defined [8]. When the system states reach the sliding manifold from arbitrary initial states, the system states begin to be confined on the manifold and the system dynamics is decided based on the sliding manifold. In addition, if the matching condition is satisfied, the sliding mode control is robust to the model uncertainty [3], parameter variation and disturbance. Therefore, the sliding mode control becomes stable after the ideal sliding mode begins to operate on the stable manifold. Assume that the sliding manifold σ(x) : $R^n \rightarrow R^m$ is a linear function as given below.

$$\sigma(x)=sx=\sigma_n x_n+\sigma_{n-1} x_{n-1}+---+\sigma_1 x_1+\sigma_0 \qquad (25)$$

Where S Є R^{mxn} and the coefficients of σ_0, σ_1---- σ_n should be chosen so as to make the sliding manifold to be stable. Next, the sliding mode control which satisfies the reaching condition is to be determined. For reaching condition, the Lyapunov function can be defined as

$$V=1/2\ \sigma^T\sigma \qquad (26)$$

If the reaching condition is satisfied, sliding mode control guarantees the stability and it confines the states on the sliding manifold. Furthermore, the derivative of the Lyapunov function should be negative definite:

$$V^*= \sigma^T\sigma \qquad (27)$$

The trajectory of the switching function σ(x,t) can be given as,

$$\log_{\sigma\rightarrow o+} \dot{\sigma} < 0$$

The configuration of the sliding surface is illustrated in Figure 3, which indicates that the arbitrary initial condition reaches the sliding manifold in finite time. During the reaching phase, the system states are transferred to the sliding manifold. However, since the tracking error [5] is not compensated in this region, the system becomes very sensitive. Unfortunately, the reaching condition is just a necessary condition to guarantee the ideal sliding mode. The reaching condition ensures only the asymptotic approach to the sliding manifold. After the time t_s, σ(t) lies on the sliding manifold so called sliding mode[6] As long as the sliding mode is maintained, the system becomes robust to model uncertainty and external noise.

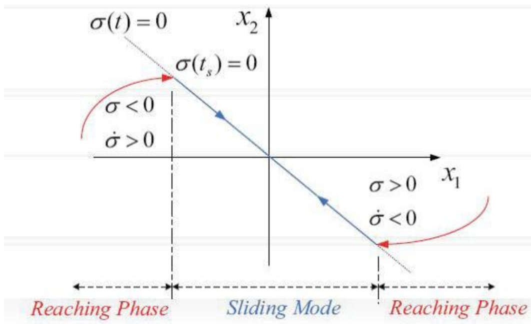

Fig 3 Configuration of ideal sliding mode

A brief overview of Utkin observer is presented here. Consider a continuous time linear systems is given by

$$x(t)= Ax(t)+Bu(t)+Ed(t) \quad (28)$$
$$y(t)= Cx(t)$$

Where $A Є R^{nxn}$, $BЄR^{nxm}$ and p≤m. Assume that the matrices *B* and *C* are of full rank and pair (*A*, *C*) is observable. Reconstructing the state variables from the measured outputs is the prime reason for the observer design. In this the observed output vector can be represented as

$$Y=C_aX_a+C_bX_b,\ X= (X_a,\ X_b) \quad (29)$$

$$C_a Є R^{px(n-p),}\ C_b Є R^{pxp,}\ \det(C_a)\neq 0 \quad (30)$$

The systems represented by the equation (8) and (15) can be written in the form

$$X_a{*}=A_{11}X_a+A_{12}Y+B_{11}U \quad (31)$$
$$Y{*}= A_{22}X_a+A_{22}Y+B_2u \quad (32)$$

The corresponding observer proposed by Utkin is given by,

$$X{*}=A_{11}x_a{*}+A_{12}y{*}+B_1U+LMsgn(y{*}-y) \quad (33)$$
$$Y{*}=A_{21}x_a{*}+A_{21}y{*}+B_2U-Msgn(y{*}-y) \quad (34)$$

Then the system error can be defined as,

$$Є_a{*}=A_{11}Єa+A_{12}Є_y+LMsgn(Є_y) \quad (35)$$
$$Є_y{*}=A_{21}Є_a+A_{22}Є_y-Msgn(Є_y) \quad (36)$$

For the large value of M, sliding motion can be induced on the output error state.

5. Simulation of results

Figures 4-9 present results of simulations of observers with additive fault and results of simulations of observers with disturbance (white noise)

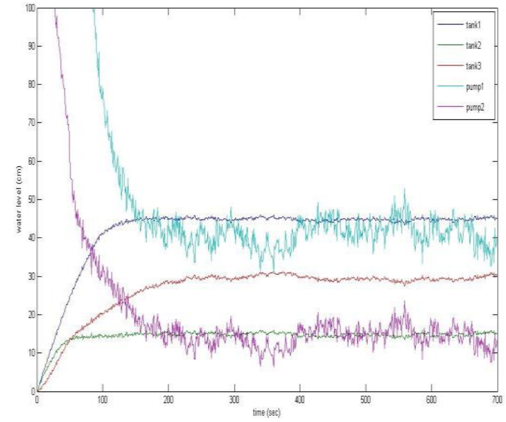

Fig 4- Residual of Luenberger observer.

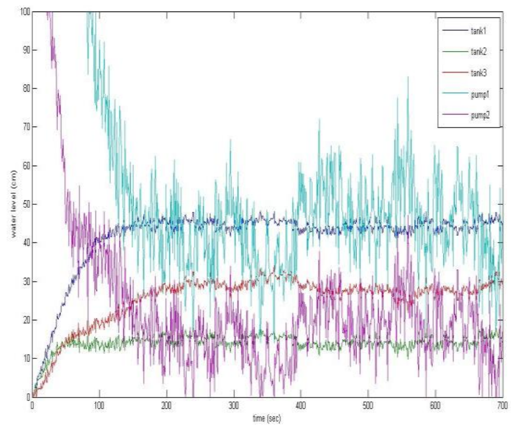

Fig 5- Residual of Luenberger observer (1)

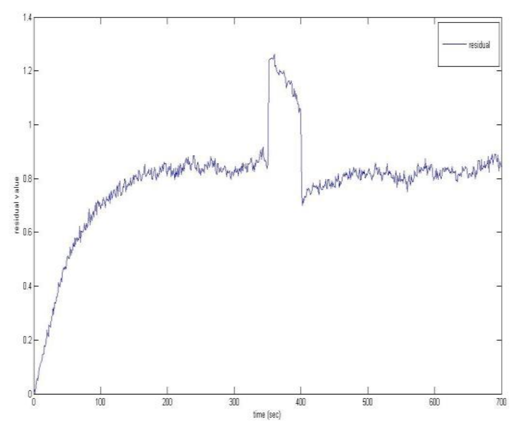

Fig 6- Residual of unknown input observer.

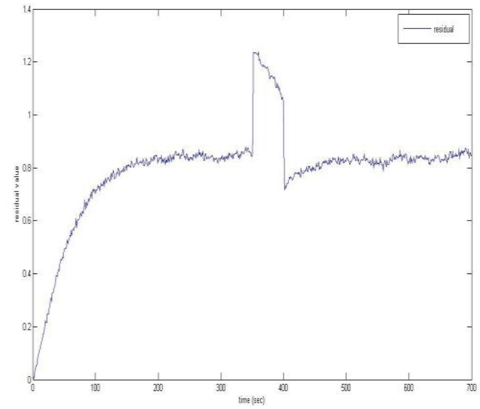

Fig 7- Residual of unknown input observer (1)

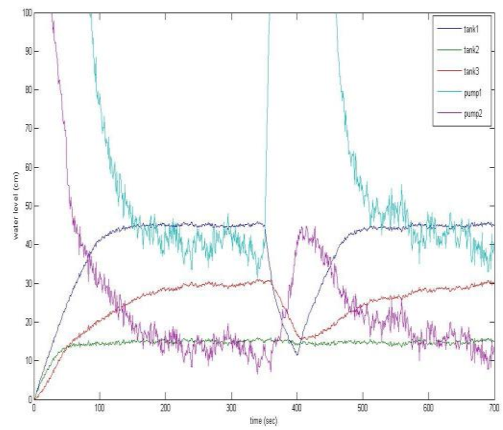

Fig 8- Residual of sliding mode observer.

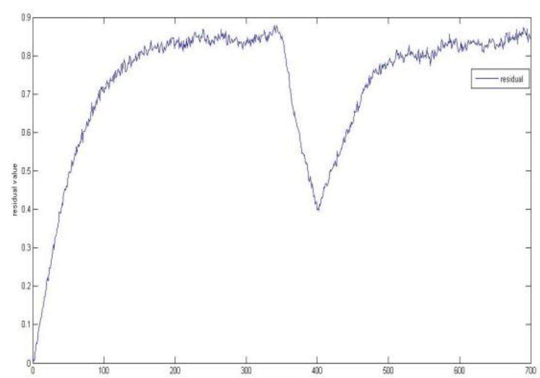

Fig 9- Residual of sliding mode observer (1)

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