Graph Coloring and its Implementation

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Abstract— Graph coloring is an important concept in graph theory. It is a special kind of problem in which we have assign colors to certain elements of the graph along with certain constraints. Suppose we are given K colors, we have to color the vertices in such a way that no two adjacent vertices of the graph have the same color, this is known as vertex coloring, similarly we have edge coloring and face coloring. The coloring problem has a huge number of applications in modern computer science such as making schedule of time table, Sudoku, Bipartite graphs, Map coloring, data mining, networking. In this paper we are going to focus on certain applications like Final exam timetabling, Aircraft Scheduling, guarding an art gallery.

Keywords— Graph, Color, Vertices, Edges.

I. INTRODUCTION OF GRAPH COLORING

Graph Coloring is one type of a Graph Labeling or you can say it is a sub branch of Graph Labeling i.e. it is a special case of it. In graph coloring we assign the labels to the elements of a graph based on some constraints or conditions. The label is actually color. In graph labeling usually we give the integer number to an edge, or vertex, or to both i.e. to an edge and to a vertex of a graph. Similarly, in graph theory, we use some colors to label the edges or vertices. But there are some restrictions on using colors. The problem is, if we have n colors, then we have to find a way for coloring vertices such that no two adjacent vertices have the same color. There exists some other graph coloring problems also, for example, Edge Coloring and Face coloring. In edge coloring, not a single vertex is connected to two edges which are having same color. And face coloring is related to Geographical map coloring. Edge coloring and face coloring problems can be transmitted to vertex coloring. A way of using colors initiated from coloring to the countries of a map. Where each surface is literally colored.

Definition of Chromatic Number: A graph G= (V, E) is k-colorable if there is exist a function c: V Æ {1, 2, … , k} (the coloring function) so that if (a,b) ÆE, then c(a) ≠ c(b) — that is, adjacent nodes must have “different colors”. The smallest number k so that G is k-colorable is called the chromatic number of G, written c(G). The lesser amount of colors needed to color a graph is known as its chromatic number. The following graph is an example of graph coloring with chromatic number. This is an example of graph coloring whose chromatic number is 3.

![Chromatic Graph](image)

The chromatic number of complete graph on ‘n’ nodes, $k_n$, has every possibly edge. Therefore its chromatic number is $c(K_n) = n$. For every tree $T$, $c(T) = 2$. The graphs, say graphs $G$ with chromatic number is 1 ($c(G) = 1$) are the graphs...
which consisting entirely isolated nodes i.e. disconnected graph— if there will single edge also, we must have chromatic number as 2 (c(G) = 2).

To find out a chromatic number of a graph is actually a NP-Complete problem. That means finding a chromatic number is belongs to NP-Complete problem category. NP-Complete problem is the part of computational theory. NP-Complete problem is a decision problem when it falls in NP and NP-Hard. We can denote the problems which are belongs to NP-Complete as NPC or NP-C.

The chromatic number for a graph G is most commonly denoted as \( \chi(G) \), but we can also denote it as \( \gamma(G) \). The chromatic number for a small graph can be computed by using Chromatic Number [G] by mathematical package Combinatory. We can use backtracking for minimum vertex coloring to compute the minimum coloring. The chromatic number of a graph should be equal to or may be greater than to its clique number. If the subgraph, say \( g \), has the chromatic number equal to the largest number of adjacent vertices in \( g \), then the given graph is Perfect Graph.

By definition, it is given that the chromatic number of the line graph \( L(G) \) and the edge chromatic number of a graph G are equal.

If a graph is having chromatic number as two, then it is known as Bi-colorable. And if chromatic number is three, then we call it as a three colorable. If we see in a general perspective, a graph which is having chromatic number as \( k \), then it is said to be the \( k \)-chromatic graph, & a graph which is having chromatic number less than or equal to \( k \) is said to be \( k \) colorable.

II. APPLICATIONS OF GRAPH COLORING

Use of graph coloring techniques in scheduling:

1. Aircraft scheduling

Accept that we are having k aircrafts, and we need to appoint them to the n flights, where the ith flight is amid the time interim of \( (a_i; b_i) \). Unmistakably, if there are two flights cover, then we can't dole out the same flying machine to both the flights. The vertices of the clash graph relates to the flights, two vertices are associated if the comparing time interims overlapped. Along these lines the clash graph is an interim graph, and which can be shaded optimally in the polynomial time.

2. Bi-processor tasks

Expect that we have a set of number of processors (machines) and a set of tasks, each one task must be executed on the two pre-assigned processors at the same time. A processor cannot chip away at 2 jobs at the same time. Case in point, such biprocessor tasks emerge when we need to calendar record exchanges between processors or on account of shared demonstrative testing of processors [2]. Consider the graph whose vertices relate to the processors, and if there is a task that must be executed on processors i and j, then we include an edge between the two relating vertices. Presently the scheduling issue can be displayed as an edge coloring of this graph: we need to relegate shades to the edges in such a route, to the point that each color shows up at most once at a vertex. Edge coloring is NP-hard [3], however there are great approximation algorithms. The most extreme degree \( \Delta \) of the graph is an evident lower bound on the quantity of colors required to shade the edges of the graph. On the other hand, if there are no multiple edges in the graph (there are no two tasks that require the same two processors), then Vizing’s Theorem gives an effective system for acquiring a \( (\Delta + 1) \) edge coloring. On the off chance that multiple edges are permitted, then the algorithm of [4] gives a 1:1-inexact arrangement.

3. Frequency assignment

Accept that we have various radio stations, distinguished by x- and y- Co-ordinates in the plane. We need to allot a frequency to each one station, however because of interferences; stations that are "close" to one another need to get diverse frequencies. Such issues emerge in frequency task of base stations in phone networks. At the outset, one may imagine that the conflict graph is planar in this issue, and the Four Color Hypothesis can be utilized, yet it is not genuine: if there are heaps of stations in little area, then they are all near one another, consequently they structure a huge coterie in the conflict graph. Rather, the conflict graph is an unit plate graph, where every vertex corresponds to a circle in the
plane with unit measurement, and two vertices are connected if and if the corresponding circles meet. A 3-approximation algorithm for coloring unit circle graphs is given in [5], yielding a 3-approximation for the frequency task issue.

4. Guarding an Art Gallery

The application of Graph Coloring additionally utilized within guarding an art exhibition. Art exhibitions along these lines need to protect their collections precisely. Amid the day the orderlies can keep a post, however during the evening this must be carried out by feature cameras. These cams are normally dangled from the roof and they pivot around a vertical hub. The pictures from the cams are sent to TV screens in the workplace of the night watch. Since it is less demanding to keep an eye on few TV screens as opposed to on a lot of people, the quantity of cams ought to be as little as possible. An extra playing point of a little number of cams is that the cost of the security framework will be lower. Then again we can't have excessively few cams, in light of the fact that all aspects of the display must be noticeable to no less than one of them. So we ought to place the cams at vital positions, such that each of them protects a substantial part of the exhibition.

![Camera](image1)

*Fig. Cameras are used for guarding an art gallery.*

In the event that we need to characterize the art display issue all the more accurately, we ought to first formalize the idea of exhibition. An exhibition is, of course, a 3-dimensional space, yet a floor arrangement provides for us enough data to place the cams. In this way we display an exhibition as a polygonal locale in the plane. We further confine ourselves to areas that are straightforward polygons, that is, districts encased by a solitary shut polygonal chain that does not converge itself. In this way we don't permit areas with gaps. A cam position in the display corresponds to a point in the polygon. A cam sees those focuses in the polygon to which it can be connected with an open fragment that lies in the inner part of the polygon.

![Simple Polygon](image2)

*Fig: A Simple Polygon.*
A triangulated basic polygon can simply be 3-colored. Subsequently, any straightforward polygon can be monitored with ⌊n/3⌋ cameras. Be that as it may maybe we can improve even. All things considered, a camera put at a vertex may protect more than simply the episode triangles. Shockingly, for any n there are straightforward polygons that require ⌊n/3⌋ cams. For a straightforward polygon with n vertices, ⌊n/3⌋ cameras are sometimes essential and constantly sufficient to have each point in the polygon obvious from no less than one.

Presently we realize that ⌊n/3⌋ cameras are constantly sufficient. Anyway we don't have a productive algorithm to compute the cam positions yet. What we need is a quick algorithm for triangulating a straightforward polygon. The algorithm ought to convey a suitable representation of the triangulation a doubly-connected edge list, for example with the goal that we can step inconstant time from a triangle to its neighbours. Given such a representation, we can compute a set of at most ⌊n/3⌋ camera positions in direct time with the system described above: utilization profundity first hun (DFS) on the double graph to compute a3-coloring and take the littlest color class to place the cameras. In the coming areas we portray how to compute a triangulation in O(nlogn) time. Suspecting this, we as of now express the last come about guarding a polygon.

5. Final Exam Timetabling as a Grouping Problem

Linear Linkage Encoding (LLE) is an as of late proposed representation plan for evolutionary algorithms. This representation has been utilized just within information grouping i.e. data clustering. Then again, it is additionally suitable for gathering issues. Last test of the year timetabling requires agreeable task of time table spaces (periods) to a set of exams. Every exam is taken by various understudies, in light of a set of constraints. In the vast majority of the studies, NE like representations are utilized. In [10], a haphazardly chose light or a substantial transformation took after by a slope climbing strategy was connected. Different combinations of constraint fulfillment procedures with hereditary algorithm sweep be found in [12]. Paqueteet. al. [14] connected a multi-objective evolutionary algorithm focused around pareto positioning with two destinations: minimize the quantity of conflicts inside the same gathering and between gatherings. Wonget. al connected a GA with a non-elitist substitution method. After hereditary administrators are connected, infringement is repaired with a slope climbing settling procedure. In their examinations a solitary issue occurrence was utilized. Ozcanet.al. [13] proposed a memetic algorithm (MA) for settling last test of the year timetabling at Yeditepe University. Mama uses an infringement administered versatile slope climber. Considering the undertaking of minimizing the quantity of exam periods and uprooting the conflicts, last test of the year time tabling decreases to the graph coloring issue [11].
III. CONCLUSION

A review is introduced particularly to extend the thought of graph coloring. Examines may get some data identified with graph coloring and its applications in computer field and can get a few thoughts identified with their field of research. This paper gives a diagram of the applications of graph coloring in heterogeneous fields to some degree however chiefly concentrates on the computer science applications that uses graph coloring concepts. Different papers focused around graph coloring have been examined identified with planning concepts, computer science applications and a review has been introduced here.

REFERENCES


