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A Simplex based re-evaluation of Goal Programming in Product Mix Optimization Project Report

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ABSTRACT

The most profitable use of a manufacturing company's scarce machine time occurs when the company is operating in the most efficient manner. The product mix problem of XYZ Company, a manufacturer of suits, jackets and trousers is used in this paper to illustrate the real-life application of the Simplex methodology of linear programming. The processing time, weekly station capacity, demand limits, selling prices and raw material costs are from a published case study. The goal is to determine the optimal level of production for each product per week in order to maximize profit. The model contains 3 decision variables, 11 constraints: 8 production stations and 3 demand limits. This was solved by using the Simplex tableau iterations and verified using the Microsoft Excel Solver. The best plan is to sell 476.23 suits, 753.75 jackets, and 600 trousers each week and the maximum weekly profit is ₹612,017.30. The demand limit for trousers is fully utilized and two stations are used: Model Preparation and Slaughterhouse. The shadow prices indicate that an additional minute at Slaughterhouse contributes approximately ₹236.31 to the weekly profit, whereas the other six stations have idle time. The results reveal that the Simplex method not only provides a production plan, but also provides a definite direction as to where additional capacity is worth purchasing.

Keywords: Linear Programming, Simplex Method, Product Mix, Shadow Price, Excel Solver, Apparel Industry.

1. INTRODUCTION

All manufacturing companies have limited resources like machine time, labour and raw material. Due to these constraints, a firm is not able to produce as much as it would like of each product. It must determine the quantities of every product to produce to maximize its profits. This decision is known as product mix problem. A bad product mix utilizes limited resources for little gain, but a good product mix utilizes the same resources for greater gain.

One popular tool of operations research for this type of decision is linear programming (LP) (Hillier & Lieberman, 2015; Winston, 2004). It formulates the objective of the firm as a linear objective function, and the constraints on the resources as linear constraints. The model is solved by the Simplex method, which was developed by Dantzig (1963). It begins with a simple attainable plan and then optimizes one step at a time from one corner of the feasible region to a better one until it reaches the best possible plan. If the model is linear, the plan obtained in this manner is the optimum plan. LP has been applied successfully in the apparel industry, for

instance, Woubante (2017) demonstrated that the use of an LP model for product mix setting in an Ethiopian garment factory could increase the monthly profit of the factory.

The apparel industry is also suitable for LP as garments go through multiple production stations including cutting, sewing, ironing and packing. Each station has a fixed number of working minutes, and each product requires a different amount of time at each station. If a product has a high selling price, it may fill up the most at a station that is already filled, and therefore may not be the best product to make. This is something that can be easily overlooked by managers who are only using their experience.

This paper examines the case of XYZ Company, a Turkish apparel maker, which was featured in a published case study (Akman & Tapsahanoglu, 2022). This company manufactures suits, jackets and trousers. It has eight stations on its production line which can be used for 2400 minutes each week and the market purchases 700 suits, 800 jackets and 600 trousers each week. The question is, how many units of a product should be produced per week to find the maximized the profit level, while not violating any of these restrictions.

The objectives of this paper are: (1) to formulate the problem as an LP model that keeps all eleven constraints; (2) to solve the model by the Simplex method and check the answer with Microsoft Excel Solver; (3) to interpret the final solution economically, using slack values and shadow prices; and (4) to compare the result briefly with the solution reported in the source study. The rest of the paper is organized as follows. The data and the method are explained in Sections 3, the model is built in Sections 4-7, the results and the Simplex solution are presented in Sections 8 and 9, the economic interpretation is presented in Section 10, and the conclusion is presented in Section 11.

2. METHODOLOGY

2.1 Data source

The data for this study is secondary quantitative data obtained from a published case study of XYZ Company, a Turkish apparel manufacturer (Akman & Tapsahanoglu, 2022). The company produces three products: suits, jackets and trousers. There are eight stations in the production. The data used are the processing time of each product at each station (Table 1), the weekly capacity of each station and the load it would carry if all demand were produced (Table 2), and the weekly demand, selling price, raw material cost and profit per unit of each product (Table 3).

Table -1: Processing time needed per unit of product (minutes).

Product	Model Prep. (min)	Slaughter-house (min)	Fusing / Labelling (min)	Sewing I (min)	Sewing II (min)	Ironing (min)	Quality Control (min)	Combining / Packaging (min)
Suits	2.05	2.01	0.99	1.54	1.23	1.84	1.50	1.40
Jackets	1.26	1.54	0.65	1.54	0.00	1.30	0.95	0.80
Trousers	0.79	0.47	0.34	0.00	1.23	0.54	0.55	0.60

Source: Akman & Tapsahanoglu (2022).

Table -2: Weekly resources held and load if all demand were produced (minutes).

Resource (station)	Unit	Held value per week	Load if all demand is produced	Spare / (shortage)
Model Preparation	min	2,400	2,917	(517)
Slaughterhouse	min	2,400	2,921	(521)
Fusing / Labelling	min	2,400	1,417	983
Sewing I	min	2,400	2,310	90
Sewing II	min	2,400	1,599	801
Ironing	min	2,400	2,652	(252)
Quality Control	min	2,400	2,140	260
Combining / Packaging	min	2,400	1,980	420

Table 2 indicates that three stations (Model Preparation, Slaughterhouse and Ironing) are unable to meet the total demand. The company therefore has to choose a product mix. The remaining five stations have sufficient demand for all the demands.

Table -3: Weekly demand and profit per unit.

Product	Weekly demand (units)	Selling price (₺)	Raw material cost (₺)	Profit per unit (₺)
Suits	700	700	220	480
Jackets	800	500	133	367
Trousers	600	250	72	178

Profit per unit = selling price – raw material cost. Labour and overhead are treated as fixed costs and are not subtracted.

2.2 Solution method

The problem is solved in 7 steps:

1. Identify the decision variables (units of suits, jackets and trousers per week).
2. Write the objective function (maximize total weekly profit).
3. Write the eleven constraints (eight stations and three demand limits) and the non-negativity conditions.
4. Add one slack variable to each constraint to reformulate the model into standard form.
5. Build the initial Simplex tableau, with all slack variables as the starting basis ($Z = 0$).
6. Update the plan: the variable with the largest positive $C_j - Z_j$ enters the basis, the minimum ratio test determines which variable to remove, and the tableau is updated. This is repeated until all $C_j - Z_j$ are zero or negative.
7. Read the production plan, the idle time and the shadow prices from the final tableau, and check the answer in Microsoft Excel Solver.

2.3 Tools and checking

The tableaus were calculated sequentially until the optimality test was satisfied; the last tableau is shown in Section 9. The same model was then entered into Microsoft Excel and solved using the Simplex LP method with Solver. Both routes gave the same answer to four decimal places.

3. MODEL FORMULATION

The problem is first expressed in words to create a mathematical model. XYZ Company wishes to select the number of suits, jackets and trousers to produce each week to maximize profit, while not exceeding the number of minutes available at each station and not producing more than the market will purchase. There are four parts to an LP model:

1. Decision variables: symbols representing the levels of the activities that the firm controls.
2. Objective function: A linear function of the decision variables to be maximized or minimized.
3. Constraints: linear restrictions imposed on the firm by its resources and its market.
4. Parameters: numbers (coefficients and right-hand sides) in the objective function and constraints.

In this case the decision variables are the weekly quantities of the three products. The objective is to maximize weekly profit. There are 11 constraints, 8 for the stations and 3 for the demand limits. The parameters are taken from Tables 1-3.

4. ASSUMPTIONS

The LP model is based on the following assumptions:

1. Certainty: the processing times, capacities, demands, prices and costs are known and do not change during the week.
2. Proportionality: If 2.05 minutes are required for Model Preparation, then 10 suits require 20.5 minutes.
3. Additivity: the total time used at a station is the sum of the time used by each product.
4. Divisibility: the quantities can be fractions, for example 476.23 suits. (Whole-unit plans are discussed in Section 8.)
5. Non-negativity: no product can be made in a negative quantity.
6. Profit per unit is the selling price less raw material cost. Labour and overhead are fixed and are not dependent on the mix.
7. Planning period is 1 week. Each station has 2400 minutes and all that is produced is sold up to the demand limit.

5. FORMULATING A LINEAR PROGRAMMING MODEL

The formulation steps are applied to XYZ Company in this section. An LP model is made of the objective function, the set of constraints and the non-negativity conditions.

Step 1 – Define the decision Variables

X_1 = number of pairs of suits made per week

X_2 = number of jackets made per week

X_3 = number of trousers made per week

Z = total profit per week (£)

Step 2 – Write the objective function

The profit per unit is $700 - 220 = 480$ for a suit, $500 - 133 = 367$ for a jacket and $250 - 72 = 178$ for a pair of trousers. The objective function is:

Maximize Z (£ profit per week) $= 480X_1 + 367X_2 + 178X_3$

Step 3 – Write the constraints

There are 2,400 minutes per week at each station. The total minutes for all products for each station shall not exceed 2,400 minutes. The demand limits prevent the model from producing more than is demanded:

1. $2.05X_1 + 1.26X_2 + 0.79X_3 \leq 2,400$ (Model Preparation)
2. $2.01X_1 + 1.54X_2 + 0.47X_3 \leq 2,400$ (Slaughterhouse)

3. $0.99X_1 + 0.65X_2 + 0.34X_3 \leq 2,400$ (Fusing / Labelling)
4. $1.54X_1 + 1.54X_2 \leq 2,400$ (Sewing I)
5. $1.23X_1 + 1.23X_3 \leq 2,400$ (Sewing II)
6. $1.84X_1 + 1.30X_2 + 0.54X_3 \leq 2,400$ (Ironing)
7. $1.50X_1 + 0.95X_2 + 0.55X_3 \leq 2,400$ (Quality Control)
8. $1.40X_1 + 0.80X_2 + 0.60X_3 \leq 2,400$ (Combining / Packaging)
9. $X_1 \leq 700$ (Demand – Suits)
10. $X_2 \leq 800$ (Demand – Jackets)
11. $X_3 \leq 600$ (Demand – Trousers)

Step 4 – Write the non-negativity conditions

$X_1, X_2, X_3 \geq 0$

Step 5 – Rewrite in standard form

The Simplex method needs equations, not inequalities. A slack variable $S_i \geq 0$ is added to each $\leq$ constraint. It is a measure of the unused portion of that resource. The model becomes:

Maximize $Z = 480X_1 + 367X_2 + 178X_3 + 0S_1 + 0S_2 + \dots + 0S_{11}$ Subject to,

1. $2.05X_1 + 1.26X_2 + 0.79X_3 + S_1 = 2,400$
2. $2.01X_1 + 1.54X_2 + 0.47X_3 + S_2 = 2,400$
3. $0.99X_1 + 0.65X_2 + 0.34X_3 + S_3 = 2,400$
4. $1.54X_1 + 1.54X_2 + S_4 = 2,400$
5. $1.23X_1 + 1.23X_3 + S_5 = 2,400$
6. $1.84X_1 + 1.30X_2 + 0.54X_3 + S_6 = 2,400$
7. $1.50X_1 + 0.95X_2 + 0.55X_3 + S_7 = 2,400$
8. $1.40X_1 + 0.80X_2 + 0.60X_3 + S_8 = 2,400$
9. $X_1 + S_9 = 700$
10. $X_2 + S_{10} = 800$
11. $X_3 + S_{11} = 600$
12. $X_1, X_2, X_3, S_1, S_2, \dots, S_{11} \geq 0$

The standard form has 14 variables (3 decision variables and 11 slack variables) and 11 equations. If $X_1 = X_2 = X_3 = 0$, then the starting plan $S_i = \text{right-hand side}$ is feasible, and the Simplex method can be started from it.

6. GENERAL FORM OF LINEAR PROGRAMMING FORMULATION

Any LP problem with n decision variables and m functional constraints can be written in the following form:

Optimize $Z = \sum c_j x_j \quad (j = 1, 2, \dots, n)$

Subject to $\sum a_{ij} x_j (\leq, =, \geq) b_i \quad (i = 1, 2, \dots, m)$

$x_j \geq 0 \quad (j = 1, 2, \dots, n)$

where c_j, a_{ij} and b_i are constants. The function being maximized or minimized is the objective function. The first m restrictions are functional (structural) constraints, while $x_j \geq 0$ are the non-negativity conditions. A feasible solution is one that meets all the constraints, while an infeasible solution violates at least one of the constraints. The feasible solution that gives the best value of Z is the optimal solution.

If there are $\leq$ constraints, the standard form includes a slack variable s_i for each constraint:

$\sum a_{ij} x_j + s_i = b_i, \quad s_i \geq 0$

The model in matrix form is to Maximize $Z = c x$, subject to $A x \leq b$ and $x \geq 0$. For XYZ Company, $n = 3$ and $m = 11$. The vector $c = (480, 367, 178)$. The vector $b = (2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 700, 800, 600)$. The first eight rows of A are the processing times in Table 1, and the last three rows are an identity matrix for the demand limits. If there is no non-basic variable that can increase Z , then the solution is optimal, that is, all $C_j - Z_j \leq 0$.

7. RESULTS & DECISIONS

The simplex solution gives the following decisions for XYZ Company:

7.1 Optimal production plan

Table -4: Optimal weekly production plan.

Product	Quantity (units per week)	Profit per unit (£)	Weekly profit (£)	Share of profit (%)
Suits	476.23	480	228,591.93	37.4
Jackets	753.75	367	276,625.37	45.2
Trousers	600.00	178	106,800.00	17.5
Total	–	–	612,017.30	100.0

Source: Simplex solution of the model in Section 6.

1. Make 476.23 suits. This is 223.77 short of the demand of 700, leaving some suit demand unmet.
2. Make 753.75 jackets. This is 46.25 below the demand of 800.
3. Make 600 trousers, the full demand.
4. Expected maximum profit: £612,017.30 per week.
5. Since the model permits fractions, a practical plan is 476 suits, 754 jackets and 600 trousers. It meets all eleven constraints and makes £611,998 per week, which is just £19.30 short of the optimum.

7.2 Resource Utilization

Table -5: Station utilization at the optimal solution.

Station	Capacity (min)	Used (min)	Idle (min)	Used (%)	Status
Model Preparation	2,400	2,400.00	0.00	100.00	Fully used
Slaughterhouse	2,400	2,400.00	0.00	100.00	Fully used
Fusing / Labelling	2,400	1,165.41	1,234.59	48.56	Not fully used
Sewing I	2,400	1,894.17	505.83	78.92	Not fully used
Sewing II	2,400	1,323.77	1,076.23	55.16	Not fully used
Ironing	2,400	2,180.14	219.86	90.84	Not fully used
Quality Control	2,400	1,760.41	639.59	73.35	Not fully used
Combining / Packaging	2,400	1,629.72	770.28	67.91	Not fully used

Source: calculated from the Excel Solver output (Table 8).

Model Preparation and Slaughterhouse are used 100%. The other six stations are idle ranging from 9.16% at Ironing to 51.44% at Fusing/Labelling. The same is true for Figure 1.

7.3 Comparison with the TOC/GP solution of the source study

The source study used the Theory of Constraints (TOC) (Goldratt & Cox, 1984) to rank products by profit per bottleneck minute, and then verified the answer with Goal Programming (GP) (Ignizio, 1976) using GAMS software, which minimizes the gap between the plan and a set of targets. Its final plan is 440.8 suits, 800 jackets and 600 trousers, reported as £612,080 per week. It is compared with the Simplex plan in Table 6.

Table -6: Source study plan versus Simplex plan.

Plan	Suits	Jackets	Trousers	Weekly profit (£)
Source study: TOC with GP (as reported)	440.80	800.00	600.00	612,080 (reported); 611,984 (recalculated)
Simplex plan (this paper)	476.23	753.75	600.00	612,017.30
Difference (Simplex – source, recalculated)	+35.43	–46.25	0.00	+33.30

Source: Akman & Tapsahanoglu (2022) for the first row; Simplex solution for the second row.

If the profit values in Table 3 are used in the source plan, the result is £611,984, which is not the same as the reported value. No plan satisfying the same eleven constraints can achieve a higher score than £612,017.30, since Simplex is able to find the true maximum of this model. These data do not allow to reach the reported £612,080. The Simplex plan earns £33.30 more than the recalculated source plan (about 0.005%). The plans vary in the split of the suit and jacket. The source plan uses all 800 jackets and has 14 minutes of Model Preparation left unused. The Simplex plan saves 46.25 jackets and 35.43 suits, and utilizes all of Model Preparation and Slaughterhouse time. TOC works one constraint at a time, and GP only confirms a target that TOC has selected, while Simplex checks all eleven constraints simultaneously. The TOC/GP result is still very close to the optimum and is easier to explain on the shop floor. The main advantage of Simplex is the extra information it gives, which is discussed in Section 10.

8. MODEL SOLUTIONS

8.1 Simplex iterations

The initial tableau has the eleven slack variables as the basis and $Z = 0$. Table 7 summarizes the pivots. The Z value of each tableau is displayed in each row, along with the variable pair selected for the next pivot. The method stopped after four pivots, at tableau 4, where every $C_j - Z_j$ is zero or negative. In tableau 4, S_9 re-enters the basis.

Table -7: Summary of the Simplex iterations.

Tableau	Z value (₹)	Entering variable	Leaving variable	Minimum ratio
0	0.00	X1 (suits)	S9 (suit demand)	700.000
1	336,000.00	X2 (jackets)	S2 (Slaughterhouse)	644.805
2	572,643.51	X3 (trousers)	S1 (Model Prep.)	376.233
3	597,472.45	S9 (suit demand)	S11 (trouser demand)	223.767
4	612,017.30	–	–	Optimal: all $C_j - Z_j \leq 0$

Entering variable = largest positive $C_j - Z_j$. Leaving variable = smallest ratio of right-hand side to pivot-column value.

8.2 Excel Solver solution

The model was entered into Microsoft Excel, with X1, X2 and X3 as the changing cells, Z as the cell to be maximized, and all eleven constraints in the constraint list. Solver used the Simplex LP method and made the variables non-negative. Table 8 shows the solution and the left-hand side (LHS) of every constraint against its right-hand side (RHS).

Table -8: Excel Solver output.

	X1	X2	X3	LHS		RHS
SOLUTION	476.23	753.75	600.00	Z = 612,017.30		
OBJECTIVE FUNCTION COEFFICIENT	480	367	178			
Constraint 1 (Model Preparation)	2.05	1.26	0.79	2,400.00	≤	2,400
Constraint 2 (Slaughterhouse)	2.01	1.54	0.47	2,400.00	≤	2,400
Constraint 3 (Fusing / Labelling)	0.99	0.65	0.34	1,165.41	≤	2,400
Constraint 4 (Sewing I)	1.54	1.54	0.00	1,894.17	≤	2,400
Constraint 5 (Sewing II)	1.23	0.00	1.23	1,323.77	≤	2,400
Constraint 6 (Ironing)	1.84	1.30	0.54	2,180.14	≤	2,400
Constraint 7 (Quality Control)	1.50	0.95	0.55	1,760.41	≤	2,400
Constraint 8 (Combining / Packaging)	1.40	0.80	0.60	1,629.72	≤	2,400
Constraint 9 (Demand – Suits)	1	0	0	476.23	≤	700
Constraint 10 (Demand – Jackets)	0	1	0	753.75	≤	800
Constraint 11 (Demand – Trousers)	0	0	1	600.00	≤	600

All eleven constraints are satisfied. Constraints 1, 2 and 11 are met as equalities, so they are binding.

8.3 Solver report

The global optimal solution report is as follows.

Objective value: 612,017.3

Infeasibilities: 0.000000

Simplex pivots (iterations): 4

Model class: LP

Decision variables: 3

Slack variables: 11

Total constraints: 11

Nonlinear variables / constraints: 0 / 0

Table -9: Variable values and reduced costs.

Variable	Value	Reduced cost
Z	612,017.30	0.000000
X1 (suits)	476.2332	0.000000
X2 (jackets)	753.7476	0.000000
X3 (trousers)	600.0000	0.000000

All three products are in the plan, so their reduced costs are zero.

Table -10: Slack or surplus and dual prices (shadow prices) of the constraints.

Row	Constraint	Slack or surplus	Dual price (£)
1	Model Preparation	0.000000	2.4504
2	Slaughterhouse	0.000000	236.3069
3	Fusing / Labelling	1234.5932	0.000000
4	Sewing I	505.8296	0.000000
5	Sewing II	1076.2332	0.000000
6	Ironing	219.8591	0.000000
7	Quality Control	639.5900	0.000000
8	Combining / Packaging	770.2755	0.000000
9	Demand – Suits	223.7668	0.000000
10	Demand – Jackets	46.2524	0.000000
11	Demand – Trousers	0.000000	65.0000

Dual price = value of Z_j under the slack variable of the constraint in the final tableau (Table 11).

8.4 Final Simplex tableau

Tableau 11 is the final (optimal) tableau. The basic variables are X_3 , X_2 , S_3 , S_4 , S_5 , S_6 , S_7 , S_8 , X_1 , S_{10} and S_9 . Model Preparation, Slaughterhouse and the trousers demand limit are binding because they are non-basic variables. All $C_j - Z_j$ values are non-negative, so no change in the plan can decrease Z , and the solution is optimal.

Table -11: Final (optimal) Simplex tableau.

Basis (C_j)	x_1	x_2	x_3	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	RHS
C_j	480	367	178	0	0	0	0	0	0	0	0	0	0	0	
x_3 (178)	0	0	1	0	0	0	0	0	0	0	0	0	0	1	600
x_2 (367)	0	1	0	-3.22	3.28	0	0	0	0	0	0	0	0	1	753.75
S_3 (0)	0	0	0	-0.35	-0.14	1	0	0	0	0	0	0	0	0	1234.59
S_4 (0)	0	0	0	1.16	-1.95	0	1	0	0	0	0	0	0	0	505.83
S_5 (0)	0	0	0	-3.03	2.48	0	0	1	0	0	0	0	0	0	1076.23
S_6 (0)	0	0	0	-0.35	-0.56	0	0	0	1	0	0	0	0	0	219.86
S_7 (0)	0	0	0	-0.64	-0.09	0	0	0	0	1	0	0	0	0	639.59
S_8 (0)	0	0	0	-0.88	0.20	0	0	0	0	0	1	0	0	0	770.28
x_1 (480)	1	0	0	2.47	-2.02	0	0	0	0	0	0	0	0	-1	476.23
S_{10} (0)	0	0	0	3.22	-3.28	0	0	0	0	0	0	0	1	-1	46.25
S_9 (0)	0	0	0	-2.47	2.02	0	0	0	0	0	0	1	0	1	223.77
Z_j	480	367	178	2.45	236.31	0	0	0	0	0	0	0	0	65	$Z = 612,017.30$
$C_j - Z_j$	0	0	0	-2.45	-236.31	0	0	0	0	0	0	0	0	-65	

Values are rounded to two decimals. $Z = £612,017.30$. S_{10} is the slack of the jacket demand limit and S_{11} the slack of the trousers demand limit.

9. ECONOMIC INTERPRETATIONS

9.1 Where the profit comes from

Jackets give the largest part of the profit (£276,625, 45.2%), followed by suits (£228,592, 37.4%) and trousers (£106,800, 17.5%). Suits have the highest profit per unit (£480), but they also use the most time at the two full stations, so they are not made up to their demand.

9.2 Shadow prices

The shadow price is the additional profit that can be obtained when one additional unit of a scarce resource is available. It is read from the Z_j row of the last tableau. If a constraint is idle, its shadow price is zero because there is no need to increase its capacity.

1. Slaughterhouse: £236.31 per additional minute.
2. Model Preparation: £2.45 per minute (after the first minute).
3. Trousers demand limit: £65.00 per additional pair of trousers that can be sold.
4. All other constraints (six stations, suit demand and jacket demand): £0.

An extra minute at Slaughterhouse is worth almost 100 times an extra minute at Model Preparation. The shadow prices also provide an explanation of the plan. A suit requires 2.01 minutes at Slaughterhouse and 2.05 minutes at Model Preparation. At the shadow prices these minutes are worth $2.01 \times 236.31 + 2.05 \times 2.45 \approx \text{₹}480$, which equals the profit of one suit. That's why suits are worth making.

9.3 How far the shadow prices can be trusted

Shadow prices are only valid for small changes. Re-solving the model shows that the ₹236.31 holds for about 14 extra minutes at Slaughterhouse, the ₹2.45 for about 20 extra minutes at Model Preparation, and the ₹65 for about 46 extra pairs of trousers. For instance, 10 additional Slaughterhouse minutes will cost approximately ₹2,363 per week. The best mix changes and the gain per minute decreases for a larger change, 100 extra minutes give ₹5,297, not ₹23,631.

9.4 Idle capacity and unmet demand

Six stations have idle time (Table 5). Only 48.56% of the students use Fusing/Labelling, 55.16% use Sewing II and 67.91% use Combining/Packaging. Ironing is used 90.84% and has 219.86 idle minutes. It is the closest of the six to its limit, but it is not binding at present, so additional Ironing capacity would not provide any profit. The time at the other stations may be utilized for training, maintenance or other work. There is also some unmet demand: 223.77 suits and 46.25 jackets. They would consume minutes at Slaughterhouse and Model Preparation that would be better spent on other products.

9.5 Decisions for management

1. First invest in Slaughterhouse capacity (extra shift, overtime or equipment), as each minute there is the most money to be made.
2. Model Preparation is considered second because it has a small gain per minute.
3. Do not invest in Ironing, Sewing, Fusing/Labelling, Quality Control or Combining/Packaging capacity at this time.
4. Look for more trousers customers, since each extra pair sold adds up to ₹65, within the range in Section 10.3.
5. Repeat the model whenever times, prices, costs or demand change, as the shadow prices will change as well.

11. CONCLUSION

The Simplex method was applied to the product mix problem of XYZ Company with all eleven constraints kept in the model. The following results were obtained:

1. The optimal weekly plan is 476.23 suits, 753.75 jackets and 600 trousers, giving a maximum profit of ₹612,017.30 per week. The same answer was obtained using Excel Solver.
2. The model preparation, slaughterhouse and the trousers demand limit are binding. The other six stations have idle time.
3. Slaughterhouse should be the first to receive any capacity investment as it adds approximately ₹236.31 to weekly profit, while Model Preparation adds only ₹2.45.
4. The additional pair of trousers that can be sold is within a small range of ₹65.
5. The Simplex plan is slightly better than the TOC/GP plan of the source study (Section 8.3) and also provides the shadow prices and slack values for decisions.

The model is a one-week plan with fixed data and the shadow prices are valid only for small changes. Despite this, it demonstrates that the Simplex method is a viable tool for apparel manufacturers: it requires only Microsoft Excel and it provides managers with information on what to produce and where additional capacity is worth purchasing.

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