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# Geometrical Approach to Kepler's Laws of Planetary Motion

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**ABSTRACT**

***We know that the earth is a planet revolving round the sun is an elliptical orbit, the sun being at the focus. The time taken by the earth to complete one revolution is called an year which is equal to 365.25 days relative to the earth the sun describes an ellipse round the earth. The elementary pen and string method to draw ellipse has been devised to examine planetary orbits on the basis of the Kepler's Laws. Besides qualitatives feature of the orbits. Quantitative depends of the orbital shape on the quantities appearing in the Kepler's Laws can also be analised with simple geometrical procedures. The method thus provides a relevant intermediate step to students prior to the study of the rigorous theory of central force problems. The students were asked questions relating to Kepler's three laws of motion, as well as what keeps planets in orbit around the sun. Less common ideas include a mix of circular and highly elliptical orbital shapes. Many students have conceptions consistent with the Kepler's second and third laws of motion and the case with which the models are adopted by students may suggests some ways to teach these concepts the types of ideas about orbital shapes and orbital behavior may originate in common depictions of orbits often seen in print and on the internet.***

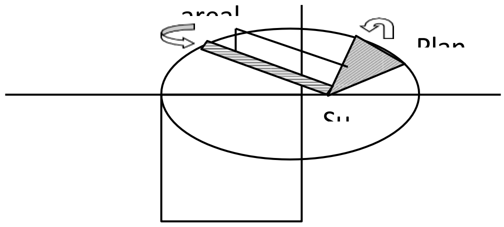
**Keywords:** *Kepler, Planet, Orbit, Focus, Ellipse*

## INTRODUCTION

Kepler's Laws of planetary motion are usually stated as Fig (i). The famous Danish Astronomer kepler discovered the following laws according to which the motion of the planets about the sun is governed.

1. The orbits of the planets are ellipse having the sun at one focci.

2. The radius vector drawn from the center of the sun to the planets sweeps over equal areas in equal times.
3. The squares of the periodic times of the various planets are proportional to the cubes of the semi-major axes of their orbits.



Semi-Major Axis Fig.(i)

*Quantities appearing in Kepler's laws of Planetary motion.*

Kepler's laws are among the most well-known scientific laws that frequently appear in articles on popular sciences. The laws require univsity level knowledge on analytical geometry to treat quadratic curves and some expertise on handling differential equations to solve physical problems if they are to be fully understood on the bases of more fundamental physical principles. i.e. Newtonian dynamics. The method has been devised by noting that the key notation of kepler's laws is the elliptic nature of planetary orbits and accordingly and simple method to handle ellipse if appropriately related to the principle of mechanics could make the laws viable even without the knowledge on university level mathematics and physics.

**REVIEW OF LITERATURE**

Kepler law is a set of laws described by Johannes Kepler(between 1609 and 1619). The law describes the planetary motion around the sun. This set of laws modified the heliocentric theory of Nicolaus copernicus by replacing it's circular orbits and epicycles with elliptical trajectories. Moreover, it explains the variation of planetary velocities.

Kepler indicated the elliptical orbit of planets by calculations of the orbit of the planet Mars using these calculations, he inferred that other planets in the solar system also have elliptical orbits. Moreover, the 2$^{nd}$ law of Kepler's law helps in establishing the theory that when a planet is closer to the sun, it can travel faster than usual. According to the third law of kepler's laws further a planet from the sun, the slower it's orbital speed becomes and vice-versa. Furthermore, kepler used his first two laws in computing the position of a planet as a function of time. This method included the solution of a transcendental equation named as the kepler's equation.

When considering the procedure of calculating the hiliocentric polar cordinates of a planet as a function of time, it includes five steps, computing the mean motion. The mean anomaly, the eccentric anomally. the true anamaly and heliocentric distance. Kepler's law describes the planetary motion around the sun where as Newton laws describes the motion of an object and it's relationship with the force that is acting on it.

**Kepler law Vs Newton law**

| | Kepler Law | Newton Law |
|---|---|---|
| Definition | Kepler law is a set of laws about planetary motion described by hohannes kepler(between 1609 and 1619) | Newton's laws are a set of three laws that describes the relationship between the motion of an object and the force action on it. |
| Explanation of objects | Planets | Any objects having a mass |
| Importance | Provided the 1st quantitative connection between the planets including the earth. | Importance as the foundation of classical mechanics one of the main branches of physics. |

Kepler's law and Newton's laws are very important in physical chemistry regarding the motion of objects. The difference between kepler and Newton law is that Kepler describes the Planetary motion around the sun whereas Newton's law describes the motion of an object and it's relationship with the force that is acting on it.

Kepler's laws of planetary motion, in astronomy and classical physics, laws describing the motions of the planets in the solar system. They were derived by the German astronomer Johannes Kepler, whose analysis of the observation of the 16th century Janish Astronomer Tycho Brahe enabled him to announce his first two laws in year 1609 and a third law nearly a decade later, in 1618, Kepler himself never numbered these laws and specially sidtinguished them from his other discoveries.

## STATEMENT OF PROBLEMS

The questions are divided into different astronomical topics, with the topic of Kepler's laws are orbits:

1. What you know about planetary orbits ans what is an orbit?
2. Why do planets move the way they do?

3. What do you know about orbit speeds. What can you tell me about the way planets move?

4. What is an ellipse?

5. Why is planet's orbit slower the farther it is from the sun?

6. Where is earth when it is travelling the fastest?

[The question in the section asked about the orbital motions of planets in the solar system and were specifically designed to be open-ended free of Jorgon and non leading]

**OBJECTIVE**

Kepler's laws improved the mdes of copericus. If they eccentricities of the planetary orbits are taken as Zero, then kepler basically aggred with copernicus.

1. The planetary orbit is circle.

2. The sun is at the center of the orbit is constant.

3. The speed of the planet in the orbit is constant.

The eccentricitites of the orbit of these planets known to copernicus and kepler are small. So the foregoing rules fibe fair approximation of planetary motion, but kepler's laws fit the observation better than does the model proposed by copernicus Kepler's correction are not all obvious

1. The planetary orbits is not a circle but ellipse.

2. The sun is not at the center but at a focal point of the elliptical orbit.

3. Neither the linear speed nor the angular speed of the planet in the orbit is constant but the speed closely linked historically with the concept of angular momentum is constant. The accentricity of the orbit of the earth makes the time from the March to the September equinox, around 179 days. A diameter would cut the orbit into equals parts, but the plane thorough the sun parallel to the equator of the earth cuts the orbit into two parts with areas in a 186 to 179 ratio, so the eccentricity of the orbit of earth is approxiamtely.

$$e \approx \frac{4186-179}{4186+179} \approx 0.015$$

**METHODOLOGY**

The most familiar and virtually unique elementary method to draw an ellipse is to use a pair of pins, a string and a pen also a paper is of course necessary on which the ellipse is drawn. The string is tied to the pins at each end and the pins are firmly pushed into the paper separately. The seperatin of the pin is thus shorter than the length of the string by using 2a long string and setting the distance of pins 2ae(e<1: orbits accentricity). One can draw an ellipse with it's two foci on the pins and having semi-major and semi-minor axes of a and b=$a\sqrt{1-e^2}$ . This method to construct an ellipse is peferred to as "Pen an string" method. Here after.

Next, we summarize some important properties of the ellipse orbit of planets moving under the gravitational potential.

$$V(r) = \frac{-k}{r}$$

Semi-major axes a of the orbit becomes

$$a = \frac{k}{2(E)}$$

Where E(<0) is the total mechanical energy. The angualr momentum vector "L" and the planet mass "m" determine semi-minor axes "b" as moving with the same one year period but with different areal velocity. Having the same orbital period. They can all be drawn with a string values of areal velocity for three orbits. Circles around the fixed pin that represents the sun. Any orbit having the particular one year period specified by the string used is drawn the corresponding circles. It is useful to prepare such data as the relation between one year period "T" and the string length 2a.

$$\text{i.e.} \quad T \infty (2a)^{\frac{3}{2}}$$

**JUSTIFICATION**

Kepler achieved much more. Firstly between 1609 and 1618. He satisfied himself that the orbit of each of the six primary planets was an ellipse with the sun at the focus. he went on to tackle the problem of the motions of planets, and their causes in Epitone Astronomiae copernicanae. Where there are some quantified references to a single planet, in addition to the main discussion which involves the planetary system. Unfortunately, Kepler's investigation of the motions was little appreciated by his contemporaries, and largely ignored subsequently. The mathematical treatment carried out in planetary motion tacked kinematically demonstrates that this angle is the uniquely appropriate foundation for a structure which is simple because it depends on orthogonality and therefore is the only workable basis for kepler's astronomy.

**ANALYSIS**

The Ecliptic: The plane of the sun's apparent path round the earth will cut the cellestical sphere in a great circle which is called the ecliptic.

Obliquity of the ecliptic: The angle between the equator and ecliptic is called obliquity of the ecliptic it is denoted by $\epsilon$ and it's value is about $23\frac{1}{2}^{0}$ approimately.

Planets: Planets are cellestial bodies moving around the sun acccording Keller's laws. There are nine major planets which in order of increasin distancce from the sun are:

i. Mercury
ii. Venus
iii. Earth
iv. Mars
v. Jupiter
vi. Saturn
vii. Uranus
viii. Neputune and
ix. pluto

Satellites: In addition to planets we have another type of cellestial bodies called satellites. The satellites move round the planets. Just as we have moon as a satellite of earth.

These three laws can be derived directly from the law of gravitation and newton's law of motion as applied to the sun and one planet but were in fact derermined earlier and helped direct Newton in his work.

**KELPER'S 1ST LAW DEDUCED FROM NEWTON'S LAW OF GRAVITATION**

Newton's Law of Gravitation:

Every particle of matter attracts every other particles of matter with a force proportional to the product of the masses of two particles concerned and inversly proportional to the square of the distance between them,

i.e. $F=\frac{GMm}{r^2}$

Where, F is th gravitaional force of attraction and G is the constant gravitation.

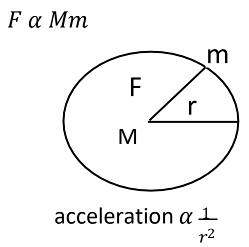
Now the acceleration of M dut ot m is $\frac{Gm}{r^2}$ and that of m due to M is $\frac{Gm}{r^2}$

Both these are in opposite direction.

Therefore, resultant acceleration is

$G\frac{(M+m)}{r^2} = \frac{\mu}{r^2}$ Where $\mu = G(M+m)$

Let, M be the mass of the sun and m be the mass of the planet. So that the attraction of the planet is towards the sun and equal to $\frac{\mu}{r^2}$ and r be the distance.




Also, from dynamics we know that equations of motion are

$$\frac{d^2r}{d\theta^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{\mu}{r^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{d\theta}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Let r=1/u

from (ii),

or, $\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0$

or, $\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0$

or, d $\left(r^2\frac{d\theta}{dt}\right) = 0dt$

or, $\int d\left(r^2\frac{d\theta}{dt}\right) = \int 0$

By integration,

or, $\left(r^2\frac{d\theta}{dt}\right) = Constant = h$

or, $\frac{d\theta}{dt} = \frac{h}{r^2}$

or, $\frac{d\theta}{dt} = hu^2$ ………………………………… $(A)$

$\therefore r = \frac{1}{u}$

or, $\frac{1}{r} = u$

or, $\frac{1}{r^2} = u^2$

From, r=$\frac{1}{u}$

or, $\frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right)$

$= \frac{u\frac{dt}{dt} - 1\frac{du}{dt}}{u^2}$

$= \frac{-\frac{du}{dt}}{u^2}$

$\therefore \frac{dr}{dt} = -\frac{1}{u^2}\frac{du}{dt}$

$= -\frac{1}{u^2}\frac{du}{dt}\frac{d\theta}{dt}$

$= -\frac{1}{u^2}\frac{du}{dt}hu^2$ …………………………………From (A)

or, $\frac{dr}{dt} = -h\frac{du}{d\theta}$ ………………………………From (B)

or, $\frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{d}{dt}\left(-h\frac{du}{d\theta}\right)$

or, $\frac{d^2r}{dt^2} = -h\frac{d}{dt}\left(\frac{du}{d\theta}\right)$

or, $\frac{d^2r}{dt^2} = -h\frac{d^2u}{d\theta^2}\frac{d\theta}{dt}$

or, $= -h\frac{d^2u}{d\theta^2}hu^2$ ...................... From (A)

or, $\frac{d^2r}{dt^2} = -h\frac{d^2u}{d\theta^2}hu^2$

or, $= \frac{d^2u}{d\theta^2} h^2u^2$

From eq. .........................(i),

or, $-h^2u^2 \frac{d^2}{d\theta^2} - h^2u^4r = \frac{-\mu}{r^2}$

or, $-h^2u^2 \frac{d^2u}{d\theta^2} - \frac{1}{u}h^2u^4 = -\mu u^2$

or, $-h^2u^2 \frac{d^2u}{d\theta^2} - h^2u^3 = -\mu u^2$

or, $h^2 \frac{d^2u}{d\theta^2} + h^2u = \mu$

or, $h^2 (\frac{d^2u}{d\theta^2} + u) = \mu$

or, $\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2}$ ........................................................... (iii)

or, $u = \frac{\mu}{h^2}[1 + ecos(\theta - \alpha)]$

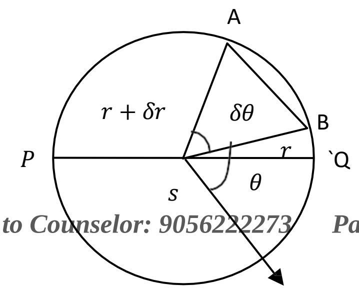
take, $h^2 = \mu l$ *and* $u = \frac{1}{r}$, *it becomes*

$\therefore \frac{1}{r} = 1 + ecos(\theta - \alpha)$

Above is polar equation of a conic whose eccentricity is e and semi-latus rectum is *l*. Since, the orbits of planets are cclosed curves, therefore, it represents an ellipse which is Kepler's 1st Law.

## DEDUCE KEPLER'S 2ND LAW OF PLANETARY MOTION FROM NEWTON'S LAW OF GRAVITATION.

Let PQ be th major axes of an ellipse and



<BSA = $\delta\theta$, then the area of <BSA from a triangle.

From Fig., we have

The equation of the area of

$$\Delta BSA = \frac{1}{2} \times SB \times SA\ Sin\partial\theta \qquad [sin\Delta\theta = \partial\theta]$$

$$= \frac{1}{2} r(r + \partial r) r\theta$$

$$= \frac{1}{2} (r^2\partial\theta + r\partial r\partial\theta) \qquad \text{[neglecting } r\partial r\partial\theta \ because\ it\ is\ very\ small]$$

$$= \frac{1}{2} r^3\partial\theta$$

$$\therefore Average\ rate\ of\ a\ sectorial\ area = \frac{Area\ of\ \Delta BSA}{rt}$$

$$= \frac{1}{2}\frac{r^2\partial\theta}{rt}$$

Total of Acctual rate of secttorial are

$$= \underset{\delta t \to 0}{lt.} \left[\frac{\frac{1}{2} r^2\partial\theta}{\partial t}\right]$$

$$= \underset{\delta t \to 0}{lt.} \left(\frac{1}{2} r^2 \frac{\partial\theta}{\partial t}\right)$$

$$= \frac{1}{2} r^2 \left(\underset{\delta t \to 0}{lt.} \frac{\partial\theta}{\partial t}\right)$$

$$= \frac{1}{2} r^2 \frac{d\theta}{dt}$$

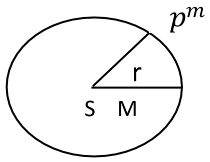
$$= \frac{1}{2}\frac{1}{u^2} hu^2$$

$$= \frac{1}{2} h$$

Which is the Kepler 2nd Law of Planetary Motion.

**KEPLER'S THIRD LAW DEDUCED FROM NEWTON'S LAW OF GRAVITATION**

Let M and m be the mas of the sun an a planets and r be the distance between them with velocity v, From Newton's law of gravitation




$$F \ \alpha \ \frac{Mm}{r^2}$$

or, $F \ \alpha \ \frac{Mm}{r^2}$

or, $F = \frac{FMm}{r^2}$

Again, Acceleration of the planet $= \frac{\mu}{r^2}$

If V be the velocity of the planet in it's orbit acceleration is $\frac{v^2}{r}$

$$\frac{v^2}{r} = \frac{\mu}{r^2}$$

or, $v^2 = \frac{\mu}{r}$ ........................................ $(i)$

Also, if T be the time period of the planet then,

$$T = \frac{2\pi r}{v}$$

or, $v = \frac{2\pi r}{T}$

or, $v = \frac{4\pi^2 r^2}{T^2}$ (Squaring we get)

or, $\frac{\mu}{r} = \frac{4\pi^2 r^2}{T^2}$

or, $\mu T^2 = 4\pi^2 r^3$

or, $T^2 = \left(\frac{4\pi^2}{\mu}\right) r^3$ (Where $\frac{4\pi^2}{\mu} = constant$)

or, $T^2 \ \alpha \ r^3$

$\therefore$ *This is the Kepler's* $3^{rd}$ *law of Planetary motion.*

**CONCLUSION**

Kepler's Laws are still valid today and have an important place in the history of science, and astronomy. They are the very step in the revolution which moved the earth centered model to the heliocentric model, and they led to the discovery of Newton's laws. Kepler's proposed the laws of orbits velocities and time periods. The law of orbit describes the trajectory of a planet which is an ellipse not a circle. We have seen that application of mechanics also provides for elliptical trajecotres only additional things is that solution of mechanics yields posibilities of other trajectoreies as well. Thus we colcude that kepler's law of orbit is consistent with Newtonian mechanics. The speed of the planet is not constant generally might have been conjectured from uniform circular motion. Rather it varies along it's path. A given area drawn to the focus is wider when it is closer to sun. it is clear that planet covers smaller are length when it is away and a larger arc length when it is closer for a given orbital area drawn from the position of the sun. It means that speed of the planet is greater at position, closer to the sun and smaller at positions away from the sun.

The planet follows a elliptical path about the sun. The sun lies at one of the foci. Gravitation force, centripetal force. Linear and angular velocities are variable with the motion. velocites are maximum at perihelion and nimimum at aphelion. Although angular velocity is variable. The angular momentum of the sytem is conserved. The expression of the total mechanical energy is same as in the case fo circular motion with the exception that semi major axis a trplaces radius r. The expression of the time period is same as in the case of circular motion with the exxception that semi major axes a replaces radius r.

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