ANALYSIS OF HEAT TRANSFER FROM FINS USING FINITE DIFFERENCE METHOD

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ABSTRACT

In this paper we present Multi-dimensional heat transfer problems can be approached in a number of ways. Analytical approach using the Laplace equation involves according the solutions of differential equations. The analysis we can get complex fin analysis depending on boundary conditions, often involving advanced mathematics using Fourier series and other special functions. A more practical approach is the use of numerical methods. The finite difference method seems to provide a good approach as using these complex problems with a variety of boundary conditions MATLAB programming. With the help of FDM method one triangular problem and a circular profile were examine.

Keywords: *Heat Transfer, Rectangular fin, Circular fin, Finite difference method.*

1. Introduction

Heat and mass transfer is one of the important fields of study for students of thermal engineering as it deals with diverse range of applications. Applications of heat and mass transfer include biological systems, common household appliances, industrial processes, electronic devices, food processing, and medicine preservation. For a thermal engineering student understanding the mechanism of heat transfer is becoming increasingly important since heat transfer plays a crucial role in the design of many appliances. Many ordinary household appliances are designed, whole or in part, by using the principles of heat transfer. The heat transfer problem can be grouped in two parts as

1. Rating Problems
2. Sizing Problems

Determination of heat transfer rate for a given temperature difference is part of problems related to rating of systems. Sizing problem deals with the determination of the size of a system in order to transfer heat at a specified rate for a specified temperature difference. there are two approaches to deal with a problem of heat transfer analysis. In the first approach one can analysis a problem experimentally by carrying out testing and taking measurements. Another approach can be analytical in which a problem or a physical situation is modeled and calculations or simulations is carried out to find the solutions. Experimental approach is close to actual physical system and can provide accurate result. However, the approach can be expensive, time consuming and some- times impracticable. With the advancement of h igh speed and low cost computing systems analytical approach seems to be more practical and inexpensive. High speed computing has resulted in various numerical approaches like Finite Difference Method (FDM) which can model a physical system more closely than idealistic mathematical model based on assumptions and approximations. Application of FDM in design of devices incorporating thermodynamics principles is increasingly becoming popular and present work is an attempt in that direction.

2. Objectives

Application of software in thermal analysis is not new. However, the cost of computation is very high in sophisticated software and their understanding demands greater skill for a proper solution. In view of the above and for deeper understanding of the software study of a numerical method and its application in heat transfer from fins is carried out in the project. The objectives of the project are

1. To understand Finite Difference Method and its application in heat transfer from fins.
2. To develop algorithms for heat transfer analysis of fins with different geometries.

3. Extended Surfaces for Heat Transfer Fin equations

The rate of hear transfer from a surface at a temperature Ts to the surrounding medium at T∞ is given by Newton's law of cooling as

$$Q_{conv} = hA_{conv}(T_s - T_\infty) \qquad (1)$$

Where, A_{conv} is the heat transfer surface area and h is the convection heat transfer coefficient. When the temperatures Ts and T∞ are fixed by design considerations, as is often the case, there are two ways to increase the convection heat transfer coefficient h or to increase the surface area A_{conv} . Increasing h may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate. The alternative is to increase the surface area by attaching to the surface extended surfaces called fins. Consider a volume element of a fin at location x having a length of Δx cross sectional area of Ac and a perimeter of p. Under steady conditions, the energy balance on this volume element can be expressed

$$Q_{cond\ ,x} = Q_{cond\ ,x+}\Delta x + Q_{cond} \qquad (2)$$

Where,

$$Q_{cond\ ,x} = Q_{cond\ ,x+}\Delta x + Q_{cond}$$

$Q_{cond\ ,x}$ = Rate of heat conduction into the element x, $Q_{cond\ ,x+}\Delta x$ = Rate of heat conduction from the element at x+Δx and

Q_{conv} = $hA_{conv}(T_s - T_\infty)$

4. Finite difference modeling of Fins

Availability of high speed computers makes it possible for today's engineer to find answers for highly complicated problems of thermodynamics and heat transfer. Though a large number of software are available for thermal analysis, it is required to understand the calculations that is carried out by the software in the background to avoid any possible pitfall. Finite difference method is one of the methods that is used as numerical method of finding answers to some of the classical problems of heat transfer. Present section deals with the fundamental aspects of Finite Difference Method and its application in study of fins.

5. Finite Difference Method (FDM)

The numerical methods for solving differential equations are based on replacing the differential equations by algebraic equations. In case of finite difference method, this is achieved by replacing the derivatives by differences. A complete Description of the method is available in

Case 1: Heat Transfer from Triangular Fins

Heat transfer from a fin of triangular cross section with length L, base thickness b and very large width w as shown in Fig.1 is considered and finite difference model of the fin is presented. The energy balance equation considering the conduction and convection can be expressed as

$$kA_{Left}\frac{T_{m-1}-T_m}{\Delta x}+kA_{Right}\frac{T_{m+1}-T_m}{\Delta x}+hA_{conv}(T_\infty-T_m)=0 \quad (3)$$

In case of triangular cross section the heat transfer areas are different for each node and using geometrical relations, as follows:

$$A_{Left}=(Height\ \times Width)_{m-\frac{1}{2}}=2w\left[L-\left(m-\frac{1}{2}\right)\Delta x\right]\tan\theta \quad (4)$$

$$A_{Right}=(Height\ \times Width)_{m+\frac{1}{2}}=2w\left[L-\left(m+\frac{1}{2}\right)\Delta x\right]\tan\theta \quad (5)$$

$$A_{conv}=(2\times Length\times Width)=2w\left(\frac{\Delta x}{\cos\theta}\right) \quad (6)$$

$$\left[1-\left(m-\frac{1}{2}\right)\frac{\Delta x}{L}\right](T_{m-1}-T_m)+\left[1-\left(m+\frac{1}{2}\right)\frac{\Delta x}{L}\right](T_{m+1}-T_m)+\frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty-T_m)=0 \quad (7)$$

$$kL_{Left}\frac{T_{m-1}-T_m}{\Delta x}+hA_{conv}(T_\infty-T_m)=0 \quad (8)$$

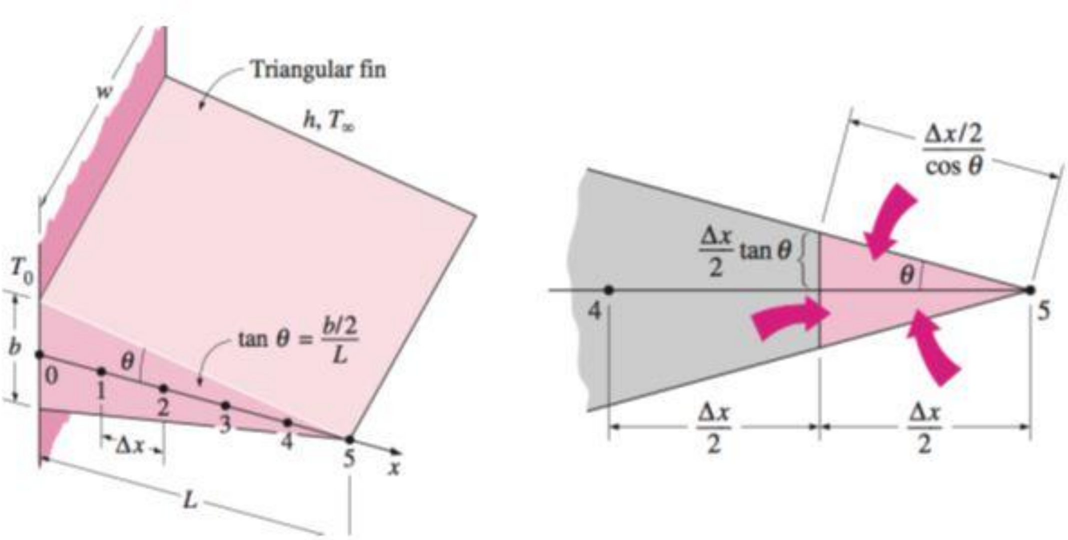

Figure [1] Triangular Fin Model

Case 2: Heat Transfer from Pin Fin

The energy balance equation considering the conduction and convection can be expressed in usual manner as

$$kL_{Left}\frac{T_{m-1}-T_m}{\Delta x}+hA_{conv}(T_\infty-T_m)=0 \quad (9)$$

In case of circular cross section the heat transfer areas are same for right and left side of the node and is equal to

$$A_{Left} = A_{Right} = A = \pi D^2/4$$

Where, D is the diameter of the pins.

Convective area is given by the

$$A_{conv} = p\Delta x$$

Where, perimeter p is p = πD

Therefore, heat transfer equation for pin fin becomes

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \quad (10)$$

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \quad (11)$$

$$kA\frac{T_{m-1} - T_m}{\Delta x} + h(\frac{p\Delta x}{2} + A)(T_\infty - T_m) = 0 \quad (12)$$

Model of a surface with pin fins is shown in Fig.2

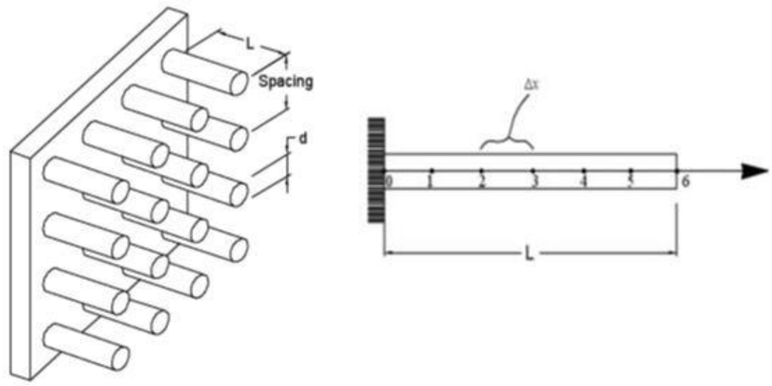

Figure [2] Triangular Fin Model

6. Analysis of Extended Surfaces for Heat Transfer in MATLAB

Based on the finite difference equations derived in previous chapter a pro- gram is developed in MATLAB ? to analyse the heat transfer from fins of various geometries. The algorithm developed for analysis is explained and several example Calculations from reference sources are taken to validate the program. The program reduces the effort required for calculation and can be used for analysis of fins with Rectangular, triangular and circular cross sections.

6.1 Development of MATLAB Code for Heat Transfer Analysis

MATLAB is a powerful computing system for handling the calculations involved in scientific and engineering problems. The name MATLAB stands for Matrix Laboratory, because the system was designed to make matrix computations particularly easy.

6.2 Algorithm for Heat Transfer Analysis of Fins

In Section xx some simple examples are presented on how to use MATLAB by entering single commands or statements at the MATLAB prompt. However, to solve problems which MATLAB can't do in one line, like calculation of a heat transfer equation (and taking all the special cases into account) a group of statements is required. A collection of statements to solve such a problem is called a program. In this section the mechanics of writing and running program is presented as an example and algorithm for heat transfer analysis of fins is presented.

Step I: Input Parameters

The heat transfer from fins depends upon the thermal property of the material, geometry of the fins surface, environment conditions which include temperature of the surrounding and temperature at the base. These are generally known in advance and are required to be entered as input to the program. Therefore, the first step of the algorithm involves input of the following parameters.

1. Thermal Properties - Thermal Conductivity (k), Convective Heat Transfer Coefficient (h)

2. Fin Geometry - Length (L), Thickness (t), Width (w)

3. Environment Conditions - Ambient Temperature ($T\infty$)

4. Fin Base Temperature (T0)

Step II: Node Specification and Nodal Spacing

Having provided the input parameters for the fin it is required to specify the number of nodes for calculation and the nodal spacing. It can also be possible to define the nodal spacing first and based on that the number of nodes can be calculated. If the number of nodes in the fin is specified to be M then the nodal spacing x becomes

$$\Delta x = \frac{L}{M-1} \qquad (13)$$

The temperature of node zero is normally given and the temperatures at the remaining nodes are to be determined. To determine them uniquely M-1 equations are needed. In order to formulate the equations the various areas are required to be calculated first

Step III: **Calculation of Heat Transfer Areas**

The heat transfer areas can be different for each node and using geometrical relations can be expressed in terms of fin geometry. Following set of areas are required for calculation of the equations.

1. Conductive area to the left of the node (A_{Left})
2. Conductive area to the right of the node (A_{Right})
3. Convective area (A_{conv})

Step IV: **Formation of Set of Algebraic Equations**

Having found the areas of heat transfer the coefficients of governing equations can be calculated. The governing equations are formulated in three parts. In part one the governing equation of base node is specified. In part two governing equations of interior nodes are calculated and finally the governing equation for boundary node based on the specified boundary condition is formulated

Step V: Solution of Algebraic Equations

Previous step produces a set of linear algebraic equations. Solving them simultaneously using the linear solve method of MATLAB gives the nodal temperature distribution.

Step VI: **Rate of Heat Transfer from Fin**

The total rate of heat transfer from the _n is simply the sum of the heat transfer from each volume element to the ambient, and for a given geometry can be determined using the relation given below.

$$Q_{fin} = \sum_{m=0}^{M-1} Q_{element,m} = \sum_{m=0}^{M-1} hA_{conv,m}(T_m - T_\infty) \quad (14)$$

Step VII: Fin efficiency

The fin efficiency is determined from the following relation.

$$\eta_{fin} = \frac{Q_{fin}}{Q_{max}} \quad (15)$$

$$Q_{max} = hA_{fin,total}(T_0 - T_\infty) \quad (16)$$

Applying the above algorithm a program was developed in MATLAB and validated using several examples cited from reference sources. A complete solution as presented by the program is demonstrated with the help of a triangular problem fin. Analysis of Heat Transfer from Fins Finite difference modelling of various fin geometries was presented in previous Chapter. Based on the models developed examples are presented in this Chapter to demonstrate the effectiveness of finite difference method in the analysis of heat transfer from fins.

Examples: Triangular Fin

Problem: An aluminium alloy fin (k=180 W/mK) of triangular cross section with length L = 5 cm, base thickness b = 1cm, and very large width w is to be considered. The base of the fin is maintained at a temperature of T0 = 200°C. The fin is losing heat to the surrounding medium at T∞= 25°C with a heat transfer coefficient of h = 15 W/m2K. Using the finite difference method with six equally spaced nodes along the fin in the x-direction, determine the temperature at the nodes.

Solution:

Input Parameters

Thermal Properties
Thermal Conductivity, k = 180 W/mK
Convective Heat Transfer Coefficient, h = 15 W/m2K
Fin Geometry
Length, L = 5 cm, Thickness, b = 1 cm, Width, w = 1 m 14
Environment Conditions
Ambient Temperature, $T\infty$ = 25°C, Fin Base Temperature, T0 = 200°C
The result given by the MATLAB Program

-0.0004

Finite Difference Equations:

-144.1507	63.0000	0	0	0
63.0000	-108.1507	45.0000	0	0
0	45.0000	-72.1507	27.0000	0
0	0	27.0000	-36.1507	9.0000
0	0	0	9.0000	-9.150

1.0e+04 *
-1.6204
-0.0004
-0.0004
-0.0004

Temperature at Nodes:

198.4198
196.8031
195.1152
193.2519
190.4802

Examples: Pin Fin

A cylindrical aluminium fin with adiabatic tip is attached to a wall with surface temperature of 300°C, and is exposed to ambient air condition of 15°C with convection heat transfer coefficient of 150 W/m2K. The fin has a uniform cross section with diameter of 1 cm, length of 5 cm and thermal conductivity of 237 W/mK Assume steady one-dimensional heat transfer along the fin and the nodal spacing to be uniformly 10 mm. Determine the nodal temperatures.

Solution by MATLAB Program:

Finite Difference Equations: **Temperature at Nodes:**

-2.0253	1.0000	0	0	0	273.5353
1.0000	-2.0253	1.0000	0	0	253.6157
0	1.0000	-2.0253	1.0000	0	239.7371
0	0	1.0000	-2.0253	1.0000	231.5480
0	0	0	1.8614	-1.8850	228.8412

7. Results and Discussion

Multi-dimensional heat transfer problems can be approached in a number of ways. Analytical approach using the Laplace equation involves finding the solutions of differential equations. The analysis can get quite complex depending on boundary conditions, often involving advanced mathematics using Bessel functions, Fourier series and other special functions. A more practical approach is the use of numerical methods. The finite difference method seems to provide a good approach as using this method a student can model fairly complex problems with a variety of boundary conditions using any software for programming. With the help of FDM method one triangular problem fin and a circular problem fin were examine and result is presented. The results are in complete agreement with the analytical solution. Nodal temperature distribution of Triangular Fin is shown in Fig. Nodal temperature distribution of Circular Fin is shown in Figure 5: Nodal Temperature Distribution Fig. 3

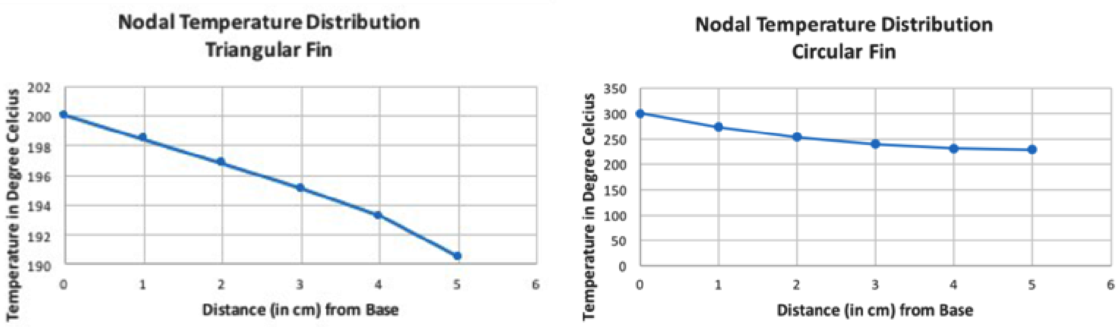

Figure [3] Nodal Temperature Distribution Triangular and Circular Fin

8. Conclusion

With the help of the FDM concept any thermal system can be modelled by applying the energy balance equation on volume element of the specified node. The FDM method was used to model three different geometries viz. rectangular, triangular and circular. A program was written in MATLAB based on the mathematical models.
F: Nodal Temperature Distribution developed in Chapter 3. The program can be used to analyse heat transfer performance of _ns having circular, triangular or rectangular cross sections. Several examples cited from reference sources are presented for validation of the program. With the help of
the program heat transfer analysis of fins can be carried out with greater degree of confidence.

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